

Electrogenic Properties of the Cloned Na⁺/Glucose Cotransporter: II. A Transport Model under Nonrapid Equilibrium Conditions

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Summary. The results of the accompanying electrophysiological study of the cloned Na⁺/glucose cotransporter from small intestine (Parent, L., Supplisson, S., Loo, D.D.F., Wright, E.M. (1992) *J. Membrane Biol.* **125**:49–62) were evaluated in terms of a kinetic model. The steady-state and presteady-state cotransporter properties are described by a 6-state ordered kinetic model ("mirror" symmetry) with a Na⁺: α MDG stoichiometry of 2. Carrier translocation in the membrane as well as Na⁺ and sugar binding and dissociation are treated as a function of their individual rate constants. Empty carrier translocation and Na⁺ binding/dissociation are the only steps considered to be voltage dependent. Currents were associated with the translocation of the negatively charged carrier in the membrane. Negative membrane potential facilitates sugar transport. One numerical solution was found for the 14 rate constants that account quantitatively for our experiment observations: i.e., (i) sigmoidal shape of the sugar-specific current-voltage curves (absence of outward currents and inward current saturation at high negative potentials), (ii) Na⁺ and voltage dependence of $K_{0.5}^{\text{sugar}}$ and $i_{\text{max}}^{\text{sugar}}$, (iii) sugar and voltage dependence of $K_{0.5}^{\text{Na}}$ and $i_{\text{max}}^{\text{Na}}$, (iv) presteady-state currents and their dependence on external Na⁺, α MDG and membrane potential, and (v) and carrier Na⁺ leak current. We conclude that the main voltage effect is on carrier translocation. Na⁺ ions that migrate from the extracellular medium to their binding sites sense 25 to 35% of the transmembrane voltage, whereas charges associated with the carrier translocation experiences 60 to 75% of the membrane electrical field. Internal Na⁺ ion binding is not voltage dependent. In our nonrapid equilibrium model, the rate-limiting step for sugar transport is a function of the membrane potential, [Na]_o and [α MDG]_o. At 0 mV and at saturating [Na]_o and [α MDG]_o, the rate-limiting step for sugar transport is the empty carrier translocation (5 sec⁻¹). As the membrane potential is made more negative, the empty carrier translocation gets faster and the internal Na⁺ dissociation becomes increasingly rate limiting. However, as [Na]_o is decreased to less than 10 mM, the rate-limiting step is the external Na⁺ ions binding in the 0 to -150 mV potential range. At 0 mV, the external Na⁺ dissociation constant K_{Na} is 80 mM and decreases to 24 mM at -150 mV. The external sugar dissociation constant K_{NaS} is estimated to be 200 μ M and voltage independent. Finally, the internal leak pathway (CNa₂ translocation) is insignificant. While we cannot rule out a more complex kinetic model, the electrical properties of the cloned Na⁺/glucose cotransporter are found to be adequately described by this 6-state kinetic model.

Key Words Na⁺/glucose cotransport · electrogenic transporters · kinetic model · nonrapid equilibrium · presteady-state kinetics

Introduction

Transport studies of cotransport mechanisms have so far been limited to steady-state analysis. The identification of the rate-limiting steps and of the voltage dependence (translocation and/or Na⁺ binding), and the values of the rate constants of the elementary reactions have yet to be resolved. The cloning and expression of membrane transport proteins has revitalized interest in transport kinetics. In the case of the intestinal brush-border Na⁺/glucose cotransporter, the expression of the clone in *Xenopus laevis* oocytes has provided unique information regarding the voltage dependence of steady-state kinetics and presteady-state currents (Umbach, Coady & Wright, 1990; Parent et al., 1991a). In addition, the Na⁺/glucose cotransporter expressed in oocytes exhibits internal Na⁺ leak currents (Umbach et al., 1990; Parent et al., 1991a). In the companion paper, we described the kinetics of the cotransporter currents as a function of extracellular Na⁺ and sugar concentrations over a large range of membrane potential. The results were presented in terms of presteady-state currents and steady-state kinetics. In this paper, we propose a simple kinetic model that accounts for the global electrical properties of the cloned Na⁺/glucose cotransporter. A single set of numerical rate constants was found to describe adequately the sigmoidal shape of the current-voltage (*I-V*) relationships, the Na⁺, sugar and voltage dependence of the steady-state kinetics parameters $K_{0.5}$ and i_{max} , the time course of the presteady-state currents and the Na⁺ carrier leak currents. While the goodness of the fit does not eliminate more complex

kinetic models, the present analysis provides insights in the mechanism of Na⁺/sugar transport which in turn casts a framework for the design of future experiments. Parts of this work have been presented earlier (Parent et al., 1991b).

Transport Model

Figure 1 shows our minimal model for the Na⁺/glucose cotransporter, i.e., with the least free parameters required to account for the cotransporter steady-state and presteady-state electrical properties. This model is described as a 6-state¹ ordered model with "mirror" symmetry with external Na⁺ ions binding first to the carrier ("last on-first off") (Segel, 1975). It consists of two transport loops linking six discrete states of the carrier. The carrier *C* possesses two binding sites for the driving ion Na⁺ and one binding site for the sugar *S* (*S* = αMDG). Phlorizin binds to the CNa₂ carrier state to form an inactive complex CNa₂Pz'. The carrier is assumed to bear a charge *z* = -2 with the result that the Na⁺-loaded carrier is electroneutral. The state transitions include three transmembrane steps: one charge-transport step *C*' ⇌ *C*'', one ternary complex translocation CNa₂' ⇌ CNa₂'' in the absence of sugar (carrier Na⁺ leak), and one fully loaded carrier translocation CNa₂S' ⇌ CNa₂S''. In addition, there are individual steps for binding of Na⁺ and sugar at each side of the membrane. The transport model is ordered in that two Na⁺ ions bind to the carrier before sugar can bind.² The current is carried by the empty form of the carrier (*C*' ⇌ *C*'') while translocation CNa₂' ⇌ CNa₂'' and CNa₂S' ⇌ CNa₂S'' are electrically silent. Membrane surface charges and unstirred layer effects at the membrane boundaries (Roomans & Borst-Pauwels, 1978) were assumed to be negligible, i.e., the substrate and cosubstrate concentrations were considered uniformly distributed between the membrane surface and the bulk aqueous phase. Each reaction step is a first or pseudo first-order process. Both Na⁺ binding and carrier translocation may be voltage dependent. Rapid equilibrium was not assumed, and so each of the seven reaction steps is treated as a function of its individual rate constants.

Rationale

The minimal model that can be proposed to describe the properties of the Na⁺/glucose cotransporter is a 4-state model with only two translocation steps: *C*' ⇌ *C*'' and CNa₂S' ⇌ CNa₂S'' and equilibrium binding of Na⁺ and sugar. Two Na⁺ ion binding sites were inferred from the coupling coefficient *n* of 2 Na⁺ : 1 αMDG

¹ The number of 6 states refer to the number of states that actually contribute (directly) to sugar transport. The 7th state is the phlorizin-bound CNa₂Pz' complex which is not translocated (inert state) and was not included in our simulations.

² In our model, the binding of two Na⁺ ions to the carrier is described as a single reaction step. This may imply that binding of the first Na⁺ ion to the carrier is in rapid equilibrium (faster than any other rate) and that binding of the second Na⁺ ion to the carrier is rate limiting. An alternative explanation is that there are two identical binding sites for Na⁺ ions with similar rate constants.

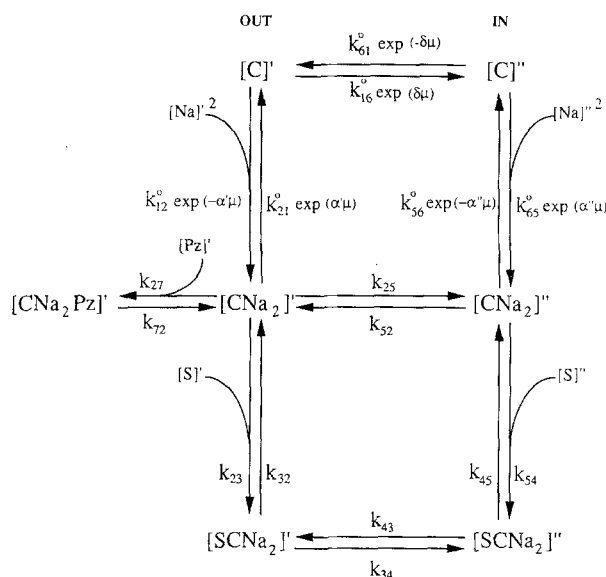


Fig. 1. Transport model for the Na⁺/glucose cotransporter. Electrical properties of the cotransporter are described by a 6-state transport model with the empty carrier bearing two negative charges. Each carrier state was identified by a number from 1 to 6, and external and internal faces of the membrane were represented by *C*₁ = *C*', *C*₂ = CNa₂', *C*₃ = CNa₂S', *C*₄ = CNa₂S'', *C*₅ = CNa₂'', *C*₆ = *C*''. Rate constants *k*₁₂, *k*₂₁, *k*₁₆, *k*₆₁, *k*₅₆, and *k*₆₅ are described by voltage-independent values (*k*_{ij}⁰) modulated by potential and/or ligand concentration (see Eqs. (12) to (25) and the Appendix). Phlorizin binds to the Na⁺-loaded carrier CNa₂' to form the inactive CNa₂Pz' complex. Phlorizin inhibits sugar transport in the presence of external Na⁺ with a *K*_i of 10 μM reflected by the rate constants *k*₂₇ and *k*₇₂. It is not yet known if phlorizin can bind to the Na⁺-loaded form CNa₂'' of the carrier. The loaded carrier is neutral and therefore not influenced by the membrane potential. α' and α'' are phenomenological constants that describe the fraction of the electrical field sensed by the Na⁺ binding to its external and internal sites, respectively. δ is the fraction of the electrical field sensed by the empty ion binding site on the carrier during membrane translocation with α' + α'' + δ = 1. μ is the electrochemical potential *FV*/*RT*. The sugar transport occurs only in presence of two Na⁺ ions that bind first to the empty carrier.

obtained from the relationship between carrier currents and [Na]_o (see Hill plot Fig. 5, Parent et al., 1991a).

The observation of phlorizin-sensitive Na⁺-dependent currents in the absence of sugar (Umbach et al., 1990; Parent et al., 1991a) suggests that translocation of CNa₂ complexes requires a third membrane translocation step. On the contrary, sugar-dependent steady-state currents were not observed in the absence of external Na⁺ (choline replacement) which indicates that formation and translocation of the CS complex is negligible in the absence of [Na]_o. The absence of sugar-dependent current in the absence of [Na]_o is also supported by irradiation-inactivation studies performed on brush-border membrane vesicles (BBMV) which have shown that Na⁺-independent sugar transport is insensitive to radiation. This suggests that Na⁺-independent sugar transport measured in BBM is unlikely to be mediated by the cotransporter (Stevens et al., 1990).

Both Na⁺ binding and carrier translocation may be voltage dependent which is implied by the voltage dependence of both $K_{0.5}^{\text{Na}}$ and $i_{\text{max}}^{\text{Na}}$ (Fig. 6, Parent et al., 1991a). The absence of voltage dependence of $K_{0.5}^{\text{MDG}}$ observed at saturating [Na]_o (Fig. 4, Parent et al., 1991a) indicates that sugar binding under these conditions is not voltage dependent.

The empty Na⁺/glucose cotransporter bears two negative charges. It has already been postulated that the Na⁺/glucose cotransporter is negatively charged in the absence of Na⁺ ions (Aronson, 1978; Kessler & Semenza, 1983; Restrepo & Kimmich, 1985b; Lapointe, Hudson & Schultz, 1986; Schultz, 1986). The first evidence was obtained by Aronson (1978) who reported that phlorizin binding to renal BBMV's was voltage dependent.³ These results were also confirmed by Turner and Silverman (1981).

We chose an ordered binding model in which two Na⁺ ions bind to the cotransporter prior to sugar association. This means that sugar binding is only possible in the presence of CNa₂. The choice of an ordered binding model over a random binding model was made to keep the model as simple as possible.⁴ However, an ordered binding mechanism is compatible with the large allosteric effect of [Na]_o observed experimentally for $K_{0.5}^{\text{MDG}}$ (Fig. 4, Parent et al., 1991a) and would explain the voltage dependence of $K_{0.5}^{\text{MDG}}$ at low [Na]_o as a consequence of the voltage dependence of Na⁺ binding to the carrier.

There were previous reports of ordered binding mechanism describing the kinetic properties of the Na⁺/glucose cotransporter in rabbit intestinal BBMV (Hopfer & Groseclose, 1980; Kessler & Semenza, 1983; Kaunitz & Wright, 1984) and in chicken intestinal cells (Restrepo & Kimmich, 1985a). An ordered binding mechanism was also suggested by reports from Peerce and Wright (1984, 1987) of a Na⁺-induced conformational change preceding sugar binding to the Na⁺/glucose cotransporter.

Na⁺ and sugar rapid equilibrium binding is not assumed. Individual rate constants were used to describe each and every reaction step including sugar and Na⁺ ions binding (Fig. 1). Because it reduces the complexity of the algebra required to model cotransport activity, the rapid equilibrium assumption has been the most enduring working hypothesis in cotransporter kinetic analysis (see for instance, Goldner, Schultz & Curran, 1969; Jauch & Läuger, 1986). Although it appears to be almost a universal assumption, exceptions can be found. For instance, Hopfer and Groseclose (1980) have proposed a kinetic model for the Na⁺/glucose cotransporter in small intestine where the rate constant for translocation is at least six times slower than the dissociation of Na⁺ ions at the extracellular interface. In addition, the rapid equilibrium assumption has been periodically questioned (e.g., Sanders et al., 1984; Sanders 1986; Schultz, 1986; Wierzbicki, Berteloot & Roy, 1990).

The density of cotransporters (C_T) was estimated to be around 50,000 carriers μm^{-2} (5×10^{10} carriers oocyte⁻¹). In a previous study, the Na⁺/glucose cotransporter density in *Xenopus* oocytes was estimated to be between 50,000 and 200,000 carriers μm^{-2} (Umbach et al., 1990). This carrier density is similar to the estimated cotransporter density in intestinal brush-border membranes (Peerce & Wright, 1984) and similar to the reported surface density of other membrane proteins such as 10,000 μm^{-2} for ACh nicotinic receptors and 50,900 μm^{-2} for the *Halobacterium* rhodopsin (Hille, 1984).

³ This explanation is valid only if one assumes the phlorizin-bound complex is not translocated.

⁴ With a 8-state random binding model, the number of King Altman patterns increases from 15 to 50 with a total of 300 terms.

FORMAL DESCRIPTION

The variation of each carrier state as a function of time is described by the difference of the forward and backward reactions. To simplify the writing, $C_1 = C'$, $C_2 = \text{CNa}_2$, $C_3 = \text{CNa}_2\text{S}'$, $C_4 = \text{CNa}_2\text{S}''$, $C_5 = \text{CNa}_2'$ and $C_6 = C''$. The set of differential equations with $n = 2$, is the following:

$$dC_1/dt = (k_{21}C_2 + k_{61}C_6) - (k_{12} + k_{16})C_1 \quad (1)$$

$$dC_2/dt = (k_{12}C_1 + k_{32}C_3 + k_{52}C_5) - (k_{21} + k_{23} + k_{25})C_2 \quad (2)$$

$$dC_3/dt = (k_{23}C_2 + k_{43}C_4) - (k_{32} + k_{34})C_3 \quad (3)$$

$$dC_4/dt = (k_{34}C_3 + k_{54}C_5) - (k_{45} + k_{43})C_4 \quad (4)$$

$$dC_5/dt = (k_{45}C_4 + k_{65}C_6 + k_{25}C_2) - (k_{54} + k_{52} + k_{56})C_5 \quad (5)$$

$$dC_6/dt = (k_{16}C_1 + k_{56}C_5) - (k_{61} + k_{65})C_6. \quad (6)$$

The total concentration of carrier in the membrane is constant:

$$C_T = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 \quad (7)$$

therefore

$$dC_1/dt + dC_2/dt + dC_3/dt + dC_4/dt + dC_5/dt + dC_6/dt = 0. \quad (8)$$

The steady-state concentrations of individual carrier states (C_1, C_2, \dots) were solved using the King and Altman (1956) diagrammatic method (see Appendix). The King-Altman method allows the concentration of any given carrier state C_x ($x = 1, 2, \dots, 6$) to be written solely in terms of rate constants and C_T . The concentration of C_x is given by the sum of 15 King-Altman (KA) patterns, each one being the product of five different rate constants. When C_x is expressed as a King-Altman term (KAT), it is denoted as KAT_i^j . The steady-state fraction of a given carrier state C_x is given by:

$$\frac{C_x}{C_T} = \frac{\sum_{j=1}^{15} \text{KAT}_x^j}{\sum_{i=1}^6 \sum_{j=1}^{15} \text{KAT}_i^j} \quad (9)$$

where x is a given carrier state, i is a carrier state (1 to 6) and j is a King Altman pattern (1 to 15). The rate equations for each carrier state are given in details in Appendix.

In a 6-state model with three membrane translocation steps, the general equation for the steady-state sugar-dependent carrier current is given by the sum of all possible membrane translocation steps:

$$i = -F[z(k_{16}C_1 - k_{61}C_6) + (z + n)(k_{25}C_2 - k_{52}C_5) + (z + n)(k_{34}C_3 - k_{43}C_4)] \quad (10)$$

where z is the cotransporter valence (electrical charge of the empty ion binding sites) associated with the empty carrier ($z = -2$), n is the number of Na⁺ ions required for sugar transport, and F is the Faraday number. Assuming that $z = -n = -2$ and including the effect of potential, Eq. (10) then becomes Eq. (11):

$$i = 2F[C_1 k_{16}^0 \exp(-\delta\mu) - C_6 k_{61}^0 \exp(\delta\mu)]. \quad (11)$$

This equation describes the steady-state carrier-mediated current at any given membrane potential (see Eqs. (16)–(17)).

Experimentally, steady-state currents are being measured at ≥ 75 msec. Sugar-dependent currents are the difference in the current value measured at steady state before and after the addition of sugar to the bath.

EFFECT OF POTENTIAL AND LIGAND CONCENTRATION

The effect of potential on Na⁺ binding and dissociation reaction steps and on the empty carrier translocation is described according to the Eyring theory of reaction rates. Sugar transport may be described as a series of activated processes in which Na⁺ ions and sugar move from a binding site across an energy barrier to the cytoplasmic site. Assuming the existence of a single symmetrical Eyring barrier in the membrane which is modulated by the fraction of the membrane electrical field sensed by each voltage-dependent step, the general form of the rate constants for any z and n values are given below. The rate constants with $z = -n = -2$ are shown at the right.

$$k_{12} = k_{12}^0 [\text{Na}]_o^n \exp(-n\alpha'\mu/2) = k_{12}^0 [\text{Na}]_o^2 \exp(-\alpha'\mu) \quad (12)$$

$$k_{21} = k_{21}^0 \exp(n\alpha'\mu/2) = k_{21}^0 \exp(\alpha'\mu) \quad (13)$$

$$k_{36} = k_{36}^0 \exp(-n\alpha''\mu/2) = k_{36}^0 \exp(-\alpha''\mu) \quad (14)$$

$$k_{65} = k_{65}^0 [\text{Na}]_i^n \exp(n\alpha''\mu/2) = k_{65}^0 [\text{Na}]_i^2 \exp(\alpha''\mu) \quad (15)$$

$$k_{16} = k_{16}^0 \exp(-z\delta\mu/2) = k_{16}^0 \exp(\delta\mu) \quad (16)$$

$$k_{61} = k_{61}^0 \exp(z\delta\mu/2) = k_{61}^0 \exp(-\delta\mu) \quad (17)$$

$$k_{25} = k_{25}^0 \exp(-(z+n)\delta\mu/2) = k_{25}^0 \quad (18)$$

$$k_{52} = k_{52}^0 \exp((z+n)\delta\mu/2) = k_{52}^0 \quad (19)$$

$$k_{34} = k_{34}^0 \exp(-(z+n)\delta\mu/2) = k_{34}^0 \quad (20)$$

$$k_{43} = k_{43}^0 \exp((z+n)\delta\mu/2) = k_{43}^0 \quad (21)$$

$$k_{23} = k_{23}^0 [S]_o \quad (22)$$

$$k_{32} = k_{32}^0 \quad (23)$$

$$k_{45} = k_{45}^0 \quad (24)$$

$$k_{54} = k_{54}^0 [S]_i \quad (25)$$

α and δ are phenomenological coefficients. δ represents the fractional dielectric distance over which the carrier ion binding site moves, and α' and α'' represent the fractional dielectric distance between the external and internal Na⁺-binding sites and the membrane-solution interfaces such that $\alpha' + \delta + \alpha'' = 1$ (Läuger & Jauch, 1986). For CNa₂S and CNa₂ translocation rate constants (k_{34} , k_{43} and k_{25} , k_{52}), it is assumed that both the ion binding site and Na⁺ ions move over the same dielectric distance δ in the membrane. The contribution from additional charge translocation in the cotransporter was assumed to be negligible (Jauch & Läuger, 1986). μ is the reduced potential FV/RT where V is the membrane potential and F , R , and T have their usual meanings.

Only the reaction steps involving the binding and dissociation of Na⁺ and the translocation of the empty carrier are influenced by the potential as shown in Eqs. (12)–(25). Negative membrane potentials would favor the sugar transport (counterclockwise reaction steps).

SIMULATIONS

All steady-state parameters, I - V curves, $K_{0.5}^{\alpha\text{MDG}}$, $i_{\text{max}}^{\alpha\text{MDG}}$, $K_{0.5}^{\text{Na}}$, and $i_{\text{max}}^{\text{Na}}$, and presteady-state currents were computer generated *simultaneously* under given *cis* and *trans* conditions with numerical values for 12 out of the 14 rate constants being assigned. Rate constants k_{27} and k_{72} describing phlorizin binding to CNa₂' were

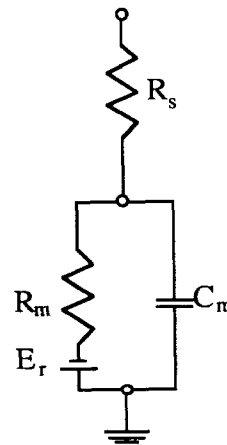
not included in the simulations but were given values that correspond to a K_D of 10 μM for phlorizin (Toggenburger, Kessler & Semenza, 1983; Ikeda et al., 1989; Kimmich, 1990).⁵

The electrical coefficients α' and δ were varied from 0 to 1 keeping $\alpha' + \delta + \alpha'' = 1$. To preserve microscopic reversibility (products of the clockwise and counterclockwise rate constants should be equal in the absence of any driving force), an internal checking procedure automatically calculates k_{52} and k_{34} from the values of all the other rate constants (*see* Appendix). Finally, simulations with the model were performed under any given Na⁺ and sugar internal (*trans*) concentration. This was required to simulate sugar transport under conditions close to our experimental conditions since sugar-dependent currents were measured in *X. laevis* oocytes with finite $[\text{Na}]_i$ that may vary from 6 to 22 mM (Dascal, 1987). Equations for the steady-state parameters ($K_{0.5}^{\alpha\text{MDG}}$, $i_{\text{max}}^{\alpha\text{MDG}}$, $K_{0.5}^{\text{Na}}$, $i_{\text{max}}^{\text{Na}}$) derived under zero-*trans* and nonzero-*trans* are given in the Appendix (Eqs. (A36) to (A43)). Briefly, under zero-*trans* conditions, the equation for the carrier current (Eq. (A20)) can be rearranged as a function of either $[\text{Na}]_o$ or $[S]_o$ and $K_{0.5}$ and i_{max} are directly determined by the product of the rate constants (Eq. (A21)). Details of the equations used for the nonzero *trans* conditions are given in the Appendix.

Computer simulations of the current time course during a pulse of potential (presteady-state currents) were carried out by integrating Eqs. (1) to (6) assuming a time interval $\Delta t = 10$ to 50 μsec . The carrier presteady-state current is the sum of all voltage-dependent reaction steps (Läuger et al., 1981; Roy et al., 1991) with:

$$i(t) = -2F\{\alpha'[k_{12}C_1 - k_{21}C_2] - \delta[C_1k_{16} - C_6k_{61}] + \alpha''[C_5k_{56} - C_6k_{65}]\} \quad (26)$$

The total membrane current was approximated as the sum of the oocyte background currents and the sugar-evoked carrier currents. Under our experimental conditions, we found that in native and in water-injected oocytes, the background currents can be modeled by an equivalent circuit composed of a series resistance (R_s) in series with a membrane resistance (R_m) and the membrane potential (E_r) both in parallel with the membrane capacitance (C_m) according to:



⁵ Phlorizin binding to the carrier forms an inactive complex CNa₂Pz' in the presence of $[\text{Na}]_o$. According to our model, the reported voltage dependence (Aronson, 1978) is secondary to the voltage dependence of C' translocation and to Na⁺ binding. Experimentally, the effect of phlorizin was to reversibly eliminate presteady-state currents and sugar-dependent currents.

where R_s is a "lump factor" that represents the sum of all possible series resistance encountered with the two-electrode voltage-clamp method. Background currents were added to the sugar-evoked carrier currents. Carrier states were considered to be in a steady state at the holding potential (e.g., -50 mV) prior to the test pulse.

As a first approximation and in the absence of any "close" initial guess, iterative simulations were preferred to a global fitting procedure. Because of the complexity of the current equation, we did not try to fit individual rate constants.

Simulations were performed until one set of rate constants was found that can globally explain our results rather than aiming at perfect multiple solutions that can account for only one result each. In addition, we did not attempt to find all possible solutions and there may be other possible solutions. For each simulation, (i) the current time course during a potential pulse in the absence and in the presence of sugar, (ii) the steady-state current-voltage relationship, and (iii) the variation of $K_{0.5}^{\text{sugar}}$, $i_{\text{max}}^{\text{sugar}}$, $K_{0.5}^{\text{Na}}$, and $i_{\text{max}}^{\text{Na}}$ as a function of applied voltage and of external cosubstrate concentration were generated simultaneously. In the initial simulations, we employed carrier translocation rate constants $\ll 1 \text{ sec}^{-1}$ and Na⁺ and sugar binding and dissociation rate constants $> 100,000 \text{ sec}^{-1} \text{ mole}^{-2}$ or $\text{sec}^{-1} \text{ mole}^{-1}$. More than 2,000 simulations were then performed during which translocation rate constants were progressively increased and most association/dissociation rate constants were decreased to account for our experimental results. Optimization of the computer-generated simulations were accomplished by progressively correcting each rate constant in order to reflect the experimental results. Numerical values for each rate constant are presented, and the range for each rate constant is discussed.

Results and Discussion

Na⁺/GLUCOSE TRANSPORT RATE CONSTANTS

Computer-generated simulations were performed with the ordered kinetic model described in Fig. 1. Figure 2 shows a set of rate constants that account for the global electrical properties of the Na⁺/glucose cotransporter: i.e., (i) the external Na⁺, the external sugar and the voltage modulation of pre-steady-state currents, (ii) the sigmoidal shape of sugar-specific current-voltage relationships, and (iii) the voltage dependence of $K_{0.5}^{\alpha\text{MDG}}$, $i_{\text{max}}^{\alpha\text{MDG}}$, $K_{0.5}^{\text{Na}}$, and $i_{\text{max}}^{\text{Na}}$ (see Figs. 3-9). The experimental results were found to be best described with $\delta = 0.7$, $a' = 0.3$, and $\alpha'' = 0$. The $\alpha'' = 0$ predicts an absence of voltage dependence for internal Na⁺ binding, and thus asymmetrical Na⁺ binding sites at the external and internal face of the membrane. Negative membrane potentials would favor the sugar transport (counterclockwise reaction steps).

MODEL PREDICTION OF THE TIME COURSE OF CARRIER CURRENTS

Current traces of cRNA-injected oocytes recorded in the presence of 100 mM [Na]_o reach a steady state

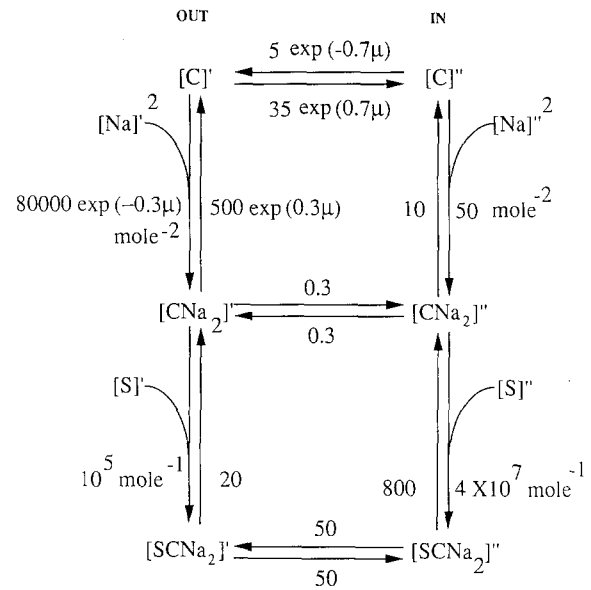


Fig. 2. Rate constants of the Na⁺/glucose cotransport kinetic model that explain our results (see Figs. 3–7). The numerical values were found using "iterative" numerical simulations of the model (Fig. 1). Units of the rate constants are either sec^{-1} , $\text{sec}^{-1} \text{ mole}^{-1}$ or $\text{sec}^{-1} \text{ mole}^{-2}$.

much slower than current traces of water-injected oocytes (Fig. 1, Parent et al., 1991a). These ON and OFF transients, distinct from the oocyte-capacitive transient, were identified as carrier presteady-state currents or carrier-gating currents. Carrier presteady-state currents were shown to be voltage dependent. They were activated by membrane depolarization from a negative holding potential. In addition, carrier presteady-state currents were not observed experimentally in the absence of [Na]_o or after the addition of either sugar or phlorizin to a Na⁺-rich extracellular medium.

Figure 3 shows the current traces obtained from an experiment (dots) performed with 100 mM [Na]_o and the current traces predicted (solid lines) with the rate constants presented in Fig. 2. The current time course simulations were performed using the oocyte-equivalent circuit with a pulse protocol identical to the one used experimentally from a holding potential of -50 mV to test potentials from -150 to $+50$ mV. As seen previously, actual current traces recorded at $+50$ mV were described by a sum of two exponential time constants of 1 and 13 msec*. Likewise, the time course of the current traces simulated at $+50$ mV can be described by two time con-

* The 1-msec time constant was identified as the oocyte-capacitive transient which was not compensated under our experimental conditions. The additional time constant of 13 msec is associated with the carrier presteady-state currents.

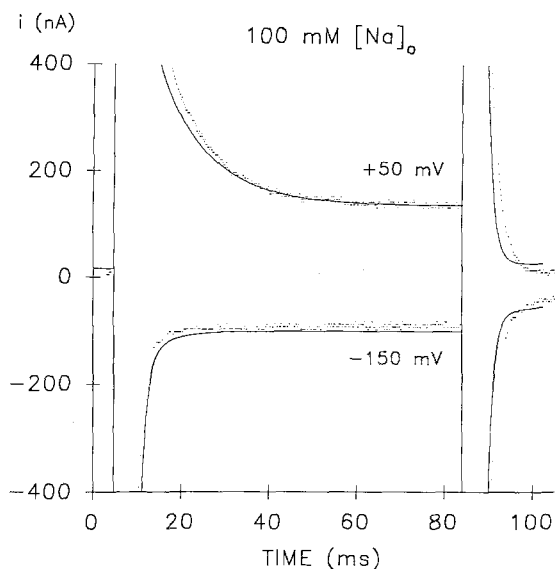


Fig. 3. Model simulation of the current time course with 100 mM $[\text{Na}]_o$. Current traces simulated in 100 mM $[\text{Na}]_o$ for $V_m = -150$ and $+50$ mV with the rate constant values shown in Fig. 2 (solid lines) were superimposed upon actual current traces (dots). The holding potential is -50 mV, and at time $t = 4.75$ msec, the potential was stepped for 80 msec to the test potential. Rate constants are shown in Fig. 2 but k_{61} was reduced from 5 to 3 sec^{-1} . The electrical circuit of the oocyte under voltage clamp is shown in the text with $R_s = 5$ k Ω , $R_m = 1.2$ M Ω , $C_m = 350$ nF, and $E_r = -65$ mV. Both the oocyte-capacitive transient ($\tau = 1$ msec) and the presteady-state carrier currents are simulated. Simulated presteady-state currents relaxed with a time constant of ≈ 15 msec when the membrane is pulsed from -50 to $+50$ mV, whereas there is no observable relaxation when the membrane is pulsed from -50 to -150 mV. In addition, the model accounts for the tail (OFF) currents when the potential is stepped back to -50 mV at 80 msec. In this figure and in Figs. 4 and 5, the data were obtained during the same experiment shown in Figs. 1 and 2 (Parent et al., 1991a), and simulations were performed with $[\text{Na}]_i = 20$ mM, $[\alpha\text{MDG}]_i = 10$ μM and a cotransporter density (C_T) of 6×10^{10} per oocyte.

stants of 1 and 13 msec. At -150 mV, current traces reach steady state with a single exponential time constant of 1 msec which is also what was observed experimentally. Tail currents measured when the potential is stepped back to the holding potential of -50 mV are also well predicted by the model. The solution of our model, therefore, mimics the ON and OFF presteady-state transient currents.

Figure 4 shows the current traces obtained in the same experiment after the addition of 1 mM $[\alpha\text{MDG}]_o$ to the bath (dots) and the current traces predicted by the model (solid lines). In the presence of sugar, the model correctly predicts both the absence of the carrier presteady-state outward currents and the increase in steady-state inward currents. In the presence of sugar, the current traces

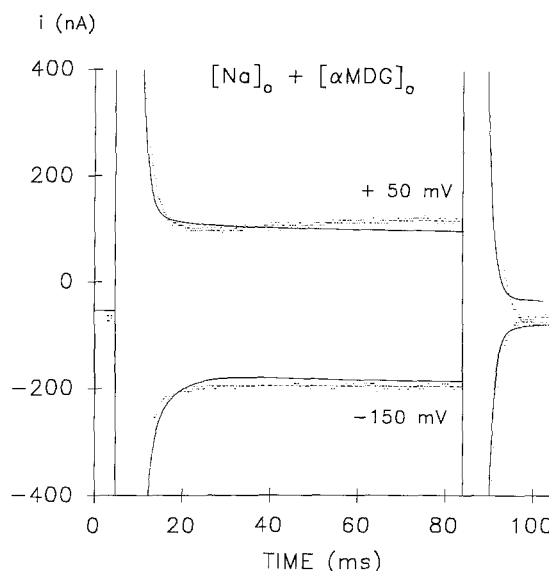


Fig. 4. Model simulation of current time course with 100 mM $[\text{Na}]_o$ and 1 mM $[\alpha\text{MDG}]_o$. Current traces simulated with 100 mM $[\text{Na}]_o$ and 1 mM $[\alpha\text{MDG}]_o$ at $V_m = -150$ and $+50$ mV (solid lines) were superimposed to actual data (dots). Same conditions as in Fig. 3. The transport model replicates the experimental observation about the absence of presteady-state currents in the presence of high $[\text{Na}]_o$ and $[\alpha\text{MDG}]_o$. Note that the steady-state carrier-mediated current measured at 75 msec is well predicted by the model (see Fig. 5).

relax to a steady state within the oocyte-capacitive time constant at both $+50$ and -150 mV.

The corresponding steady-state sugar-dependent I - V curve predicted by the model (solid line) is compared to the actual current values in Fig. 5. Both the predicted and the measured I - V curves are sigmoidal with a roughly linear segment between $+20$ and -60 mV and a plateau phase at more negative potentials (see Fig. 2, Parent et al., 1991a). The correspondence between the model prediction and the actual current values is particularly satisfying in the $+10$ to -150 mV range where the voltage dependence of the kinetic parameters $K_{0.5}^{\alpha\text{MDG}}$, $j_{\text{max}}^{\alpha\text{MDG}}$, $K_{0.5}^{\text{Na}}$ and $i_{\text{max}}^{\text{Na}}$ was measured. In addition, the model correctly predicts the absence of outward currents. The predicted carrier-mediated Na^+ leak current, which corresponds to the translocation of the CNa_2 complex, is shown as a dashed line. These current values are similar to those recorded experimentally (Umbach et al., 1990; Parent et al., 1991a).⁶

The model, therefore, correctly accounts for the voltage dependence of the presteady-state currents

⁶ Na^+ -dependent carrier currents with a phlorizin K_i of 10 μM amount to 5 to 15% of the carrier current measured in the presence of sugar (Umbach et al., 1990).

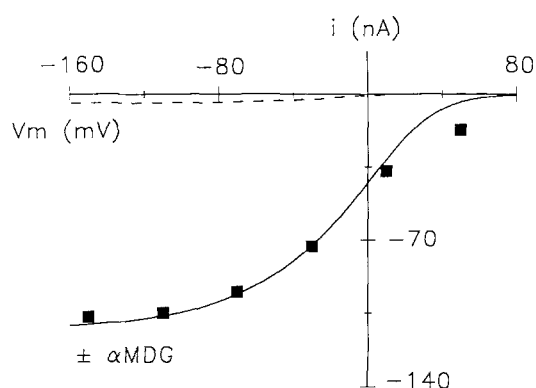


Fig. 5. Steady-state sugar-dependent I - V curves. Sugar-dependent I - V curve predicted at steady state by the model (solid line) is shown superimposed over the actual current values (\pm sugar) of the experiment shown in Figs. 3 and 4 (■). Simulations were performed with 100 mM $[\text{Na}]_o$, 1 mM $[\alpha\text{MDG}]_o$, 20 mM $[\text{Na}]_i$ and 10 μM $[\alpha\text{MDG}]_i$. The experiment was performed with 100 mM $[\text{Na}]_o$ and 1 mM $[\alpha\text{MDG}]_o$. The dashed line shows the predicted carrier-mediated Na^+ “leak” current in the absence of external sugar (for comparison, *see* Umbach et al., 1990).

recorded in the presence of 100 mM $[\text{Na}]_o$, the absence of presteady-state currents after the addition of sugar to the Na^+ -rich medium, the magnitudes of the steady-state sugar-dependent and sugar-independent currents, and the sigmoidal shape of the sugar-dependent I - V curves.

MODEL PREDICTION FOR $K_{0.5}^{\alpha\text{MDG}}$, $K_{0.5}^{\text{Na}}$, $i_{\text{max}}^{\text{Na}}$ AND $i_{\text{max}}^{\alpha\text{MDG}}$

Steady-state kinetics of the Na^+ /glucose cotransporter were reported in the experimental paper. $K_{0.5}^{\alpha\text{MDG}}$ was found to be both Na dependent and voltage dependent only at low $[\text{Na}]_o$. Figure 6A shows $K_{0.5}^{\alpha\text{MDG}}$ as predicted by the model (solid lines) superimposed upon experimental $K_{0.5}^{\alpha\text{MDG}}$ values (*see* Fig. 4A, Parent et al., 1991a). The model predicts qualitatively and quantitatively that the voltage dependence of $K_{0.5}^{\alpha\text{MDG}}$ increases as $[\text{Na}]_o$ is decreased from 100 to 2 mM. At 0 mV, $K_{0.5}^{\alpha\text{MDG}}$ is predicted to increase from 330 μM to 8 mM to 10 mM when $[\text{Na}]_o$ decreases from 100 to 10 to 2 mM.

$i_{\text{max}}^{\alpha\text{MDG}}$ was voltage dependent, especially between 0 and -100 mV, and slightly Na^+ -dependent between 10 and 100 mM $[\text{Na}]_o$ (*see* Fig. 4B, Parent et al., 1991a). $i_{\text{max}}^{\alpha\text{MDG}}$ predicted by the model (solid lines) under the same conditions is superimposed upon the $i_{\text{max}}^{\alpha\text{MDG}}$ values that were reported for 10 and 100 mM $[\alpha\text{MDG}]_o$ (Fig. 6B). Our model correctly predicts the voltage dependence of $i_{\text{max}}^{\alpha\text{MDG}}$ as measured at 100 mM $[\text{Na}]_o$ over the complete range of potential and the voltage dependence of $i_{\text{max}}^{\alpha\text{MDG}}$ as

measured at 10 mM $[\text{Na}]_o$ between -80 and -160 mV. Although there is some discrepancy for $i_{\text{max}}^{\alpha\text{MDG}}$ at 10 mM $[\text{Na}]_o$, especially in the -50 to 0 mV range, the transport model accounts for the weak $[\text{Na}]_o$ dependence of $i_{\text{max}}^{\alpha\text{MDG}}$ between 10 and 100 mM $[\text{Na}]_o$.

Both $K_{0.5}^{\text{Na}}$ and $i_{\text{max}}^{\text{Na}}$ were shown to be voltage- and sugar-dependent (*see* Fig. 6, Parent et al., 1991b). Figure 7A shows $K_{0.5}^{\text{Na}}$ as predicted by the model for 1 and 20 mM $[\alpha\text{MDG}]_o$ superimposed upon the actual data. The model accounts qualitatively for the voltage dependence of $K_{0.5}^{\text{Na}}$. The predicted $K_{0.5}^{\text{Na}}$ values at 0 mV decrease from 54 to 20 mM when $[\alpha\text{MDG}]_o$ increases from 1 to 20 mM. $i_{\text{max}}^{\text{Na}}$ simulated at 1 mM $[\alpha\text{MDG}]_o$ with the parameters of the model are shown in Fig. 7B superimposed upon experimental results.⁷ The model accounts for the voltage dependence of $i_{\text{max}}^{\text{Na}}$ at 1 mM $[\text{sugar}]_o$ and predicts a similar voltage dependence for $i_{\text{max}}^{\text{Na}}$ at 20 mM $[\text{sugar}]_o$ (*results not shown*).

PRESTEADY-STATE CURRENTS

Presteady-state currents were elicited by a step of voltage from negative to positive values (Figs. 3 and 7-10, Parent et al., 1991b). Presteady-state currents were modulated by $[\text{Na}]_o$, $[\text{sugar}]_o$ and membrane potential. They were specifically recorded in the absence of external sugar, and it is therefore likely that they arise from the translocation of the empty carrier in response to a positive pulse of potential. The description of the nature of the presteady-state currents relies heavily on the assumption that the empty carrier is negatively charged.⁸

Carrier presteady-state currents relaxed with a half time constant $\tau_2 \approx 13$ msec at $+50$ mV (*see* Figs. 3 and 8, Parent et al., 1991a). Given that only one relaxation was observed, a minimum of two slow steps along the transport cycle is required to describe the presteady-state currents (Hille, 1984; Wierzbicki et al., 1990; Roy, Wierzbicki & Sauré, 1991). Figure 8 shows the distribution of the six possible carrier states when the membrane potential is stepped from -50 to $+50$ mV and then back to -50 mV in the presence of 100 mM $[\text{Na}]_o$. In the

⁷ Note that $i_{\text{max}}^{\alpha\text{MDG}}$ and $i_{\text{max}}^{\text{Na}}$ were measured on separate experiments. Ideally, at saturating $[\text{Na}]_o$ and saturating $[\alpha\text{MDG}]_o$, $i_{\text{max}}^{\alpha\text{MDG}}$ would be equal to $i_{\text{max}}^{\text{Na}}$ when measured on the same oocyte. In our simulations, C_7 was scaled to reflect the difference in current density between the two oocytes.

⁸ The following interpretation of the carrier presteady-state currents rests entirely on the assumption that the empty carrier is negatively charged ($z \neq 0$) and δ is greater than zero (presteady-state currents correspond to the gating currents). We have not tried to find a solution with $z = 0$ or with noninteger charges ($z = 1/4, 1/2, \text{etc.}$).

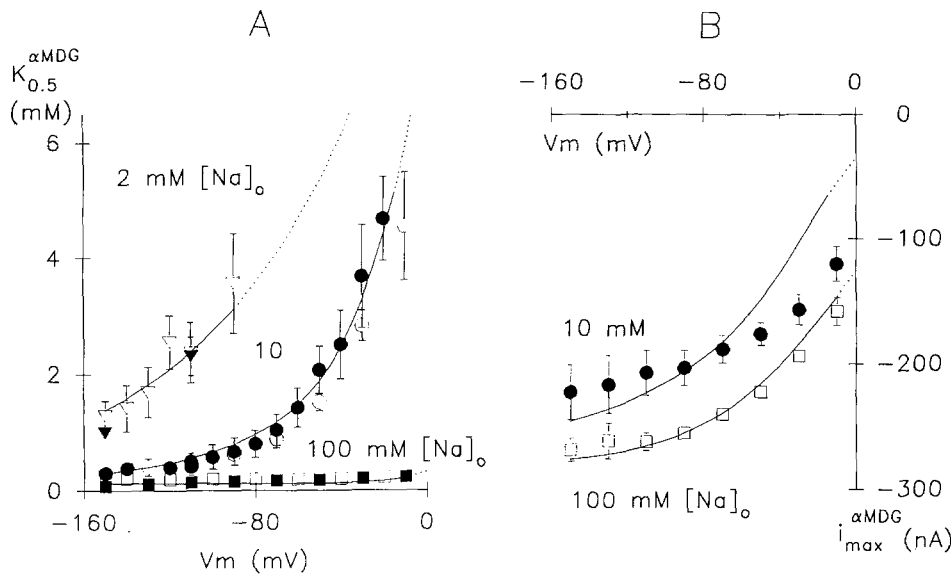


Fig. 6. Voltage and Na⁺ dependence of $K_{0.5}^{\alpha\text{MDG}}$ and $i_{\text{max}}^{\alpha\text{MDG}}$. (A) $K_{0.5}^{\alpha\text{MDG}}$ fit to experimental data using the rate constants shown in Fig. 2. $K_{0.5}^{\alpha\text{MDG}}$ simulated with transport model (solid lines) is shown superimposed to $K_{0.5}^{\alpha\text{MDG}}$ measured experimentally with 2 ($\nabla, \blacktriangledown$), 10 (\bullet, \circ) and 100 (\square, \blacksquare) mM $[\text{Na}]_o$. $K_{0.5}^{\alpha\text{MDG}}$ and $i_{\text{max}}^{\alpha\text{MDG}}$ measured at 10 (\bullet) and 100 (\square) mM $[\text{Na}]_o$ were measured on the same oocyte. The error bar represent the error on the estimation of $K_{0.5}^{\alpha\text{MDG}}$ from actual current values. $K_{0.5}^{\alpha\text{MDG}}$ simulation was performed according to Eq. (A39). The dotted lines show the model predictions beyond the experimental data. At 0 mV, $K_{0.5}^{\alpha\text{MDG}}$ would be 8 mM at 10 mM $[\text{Na}]_o$ and 10 mM at 2 mM $[\text{Na}]_o$. (B) $i_{\text{max}}^{\alpha\text{MDG}}$ fit to experimental data. $i_{\text{max}}^{\alpha\text{MDG}}$ simulated with transport model superimposed to $i_{\text{max}}^{\alpha\text{MDG}}$ was measured experimentally with 10 (\bullet) and 100 mM $[\text{Na}]_o$ (\square). $i_{\text{max}}^{\alpha\text{MDG}}$ simulation was performed according to Eq. (A37) with the rate constants shown in Fig. 2. C_7 is 15×10^{10} carriers oocyte⁻¹.

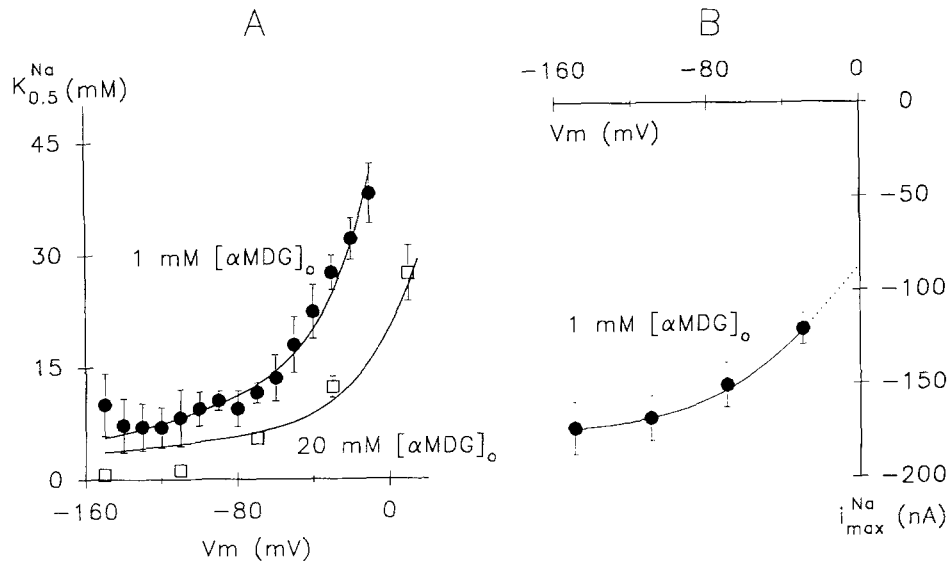


Fig. 7. Voltage and sugar dependence of $K_{0.5}^{\text{Na}}$ and $i_{\text{max}}^{\text{Na}}$. (A) $K_{0.5}^{\text{Na}}$ fit to experimental data. The error bar represents the error on the fit to the Hill equation. $K_{0.5}^{\text{Na}}$, simulated with the transport model (solid lines) are shown superimposed to $K_{0.5}^{\text{Na}}$ obtained experimentally for 1 (\bullet) and 20 mM $[\alpha\text{MDG}]_o$ (\square) in two separate experiments. $K_{0.5}^{\text{Na}}$ was simulated from Eq. (A43). (B) $i_{\text{max}}^{\text{Na}}$ fit to experimental data. $i_{\text{max}}^{\text{Na}}$ simulated with the transport (solid lines) superimposed to $i_{\text{max}}^{\text{Na}}$ was obtained experimentally for 1 mM $[\alpha\text{MDG}]_o$. The curve shows the behavior of $i_{\text{max}}^{\text{Na}}$ according to the rate constants shown in Fig. 2. $i_{\text{max}}^{\text{Na}}$ was simulated from Eq. (A41). C_7 is 10×10^{10} carriers oocyte⁻¹. The dotted lines show the model predictions beyond the experimental data.

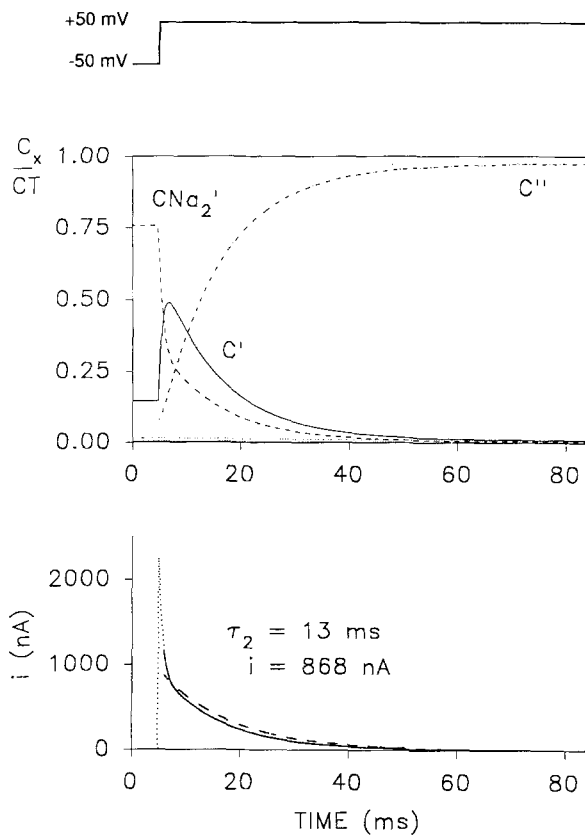


Fig. 8. Reconstruction of the carrier presteady-state current. Upper panel. Time course of carrier distribution was predicted by the model in the presence of 100 mM $[Na]_o$ when the membrane potential is stepped from -50 to +50 mV. Only three carrier states C' , CNa_2' and C'' account for >95% of all carrier states throughout the simulation. At -50 mV, the major state carrier is CNa_2' . As the membrane potential rises to +50 mV, CNa_2' is converted into C' and C'' . This series of reaction steps is responsible for the ON currents (see lower panel). Lower panel. Carrier-mediated current was reconstructed in the absence of the oocyte-capacitive transient using Eq. (26) in the presence of 100 mM $[Na]_o$. There is no steady-state carrier-mediated current at $V_h = -50$ mV (prior to the pulse). As the potential is stepped from -50 to +50 mV at 4.75 msec, carrier-mediated current rises rapidly to a peak at 1320 nA before vanishing at steady state. Under our experimental conditions, the rapid rise of the presteady-state current is likely to be obscured by the oocyte-capacitive transient (dotted line). The carrier presteady-state current can be fitted by a single exponential function (dashed line) $i = i_0 \exp(-t/\tau)$ with $t =$ time, $\tau = 13$ msec and $i_0 = 868$ nA where i_0 is the maximum current extrapolated at the beginning of the pulse (4.75 msec). For this simulation, C_T was 4×10^{10} carriers oocyte⁻¹.

absence of sugar, four carrier states are mainly present; C' , CNa_2' , C'' and CNa_2'' . As observed at the holding potential of -50 mV (prior to the onset of the pulse), 75% of the carrier is in the CNa_2' state with C' (15%) and C'' (7%) completing the carrier

distribution. This represents an asymmetrical distribution of the carrier with more carriers available from the external than from the internal face of the membrane. The distribution and the translocation of C' and C'' is controlled by the membrane potential with a negative membrane potential more likely to increase the concentration of C' . Translocation of $CNa_2' \rightleftharpoons CNa_2''$ is electrically silent (Eqs. (16)–(19)). As the potential is stepped from -50 to +50 mV (at time $t = 4.75$ msec), CNa_2' is dissociated into external Na^+ ions and C' , and then the empty carrier, returns to the internal face of the membrane (C''). At the end of the pulse, C'' represents 98% of the carrier states.⁹ As a consequence of the actual rate constants at +50 mV ($k_{21} = 911$ sec⁻¹, $k_{12} = 4.4$ sec⁻¹, $k_{16} = 141$ sec⁻¹ and $k_{61} = 0.74$ sec⁻¹), there is a transitory accumulation of C' during the pulse. The empty carrier concentration C' rises from 15% (at $V_h = -50$ mV) to peak at 45%, 2 msec after the step of potential, before vanishing at the steady state. The carrier transient current peaks at 1320 nA at the beginning of the pulse (Fig. 8, lower panel). The predicted transient current is described by a single exponential function (dashed line) with a time constant $\tau = 13$ msec which corresponds to the observed time constant for the presteady-state current. The transient outward current can, therefore, be understood as the result of the rearrangement of three carrier states $CNa_2' \rightleftharpoons C' \rightleftharpoons C''$. Rate constants k_{21} and k_{16} are the two “slow” rate constants required to generate the ON rate for the presteady-state currents. The rising phase of the carrier presteady-state currents was not observed experimentally as it is obscured by the capacitive transient of the oocyte. The OFF presteady-state current simulated as the potential is stepped back to -50 mV is also shown. The OFF rate is controlled by the value of the rate constant k_{61} in the reaction $C' \xrightarrow{k_{61}} C''$.

The two-step carrier rearrangement from -50 to +50 mV can also explain the increase of presteady-state currents with increasingly negative holding potentials, e.g., current increasing as the holding potential was made more negative (see Fig. 9, Parent et al., 1991a). Simulations with the model indeed predicts that increasing the holding potential from -50 to -200 mV before the test potential of +50 mV will also increase from 75 to 97% the number of carriers in the CNa_2' state. So when the potential is stepped to +50 mV, the transient increase in C' and the carrier-mediated currents are accordingly larger.

⁹ The backward reaction $C' \xrightarrow{k_{61}} C''$ is not favored by positive potentials. At +50 mV, the rate constant k_{61} is only 0.74 sec⁻¹. In addition, with KNa^+ between 44 and 450 mM, there is no predicted accumulation of CNa_2' species for $[Na]_i$ below 40 mM.

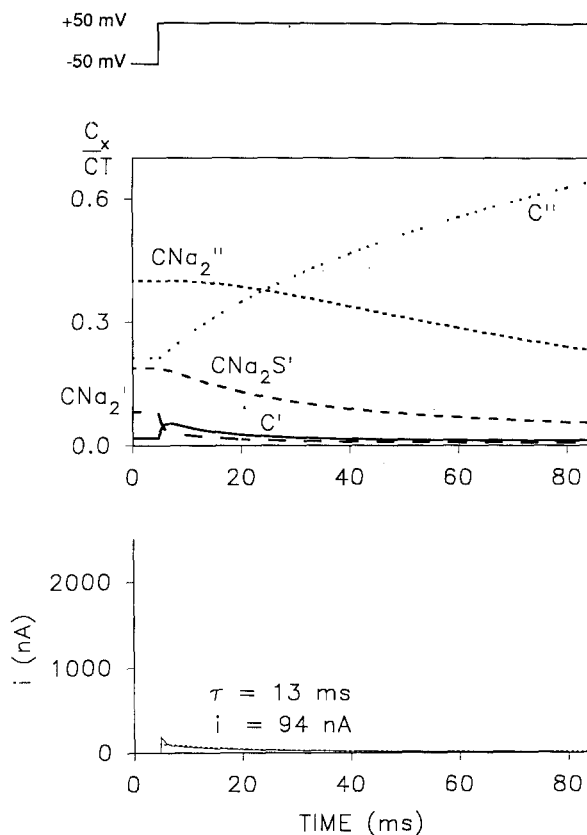


Fig. 9. Reconstruction of the carrier presteady-state current in the presence of sugar. Upper panel. Time course of carrier distribution was predicted by the model in the presence of 100 mM [Na]_o and 1 mM [αMDG]_o from -50 to +50 mV. The three major carrier states are CNa₂' (40%), CNa₂S' (20%) and C'' (21%) at -50 mV. As the potential is stepped to +50 mV, C'' rises steadily to 63% as positive membrane potential favors the accumulation of empty carriers at the internal face. Lower panel. Carrier-mediated current is reconstructed in the absence of the oocyte-capacitive transient using Eq. (11) in the presence of 100 mM [Na]_o and 1 mM [αMDG]_o. Note this time, the steady-state current of -50 nA measured at V_h = -50 mV. When the potential is stepped from -50 to +50 mV, carrier-mediated current transiently rises only to 88 nA, before reaching a steady-state value of +12 nA. Current relaxation was approximated by one exponential function with i₁ = 94 nA and τ₁ = 13 msec (dashed line). The presteady-state current generated under these conditions is 10-fold smaller than in the absence of [Na]_o but the current relaxes with the same time constant τ of 13 msec (see Fig. 8). As in Fig. 8, C_T is 4 × 10¹⁰ carriers oocyte⁻¹.

We reported that presteady-state currents were not observed after the addition of sugar to the Na-rich medium (see Fig. 7C, Parent et al., 1991a). Figure 9 presents the distribution profile of the carrier states simulated with 100 mM [Na]_o and 1 mM [αMDG]_o during a pulse of potential from -50 to +50 mV. At -50 mV and in the presence of [Na]_o and [sugar]_o, the major carrier states are CNa₂' (40%), CNa₂S' (20%) and C'' (21%) (Fig. 9, upper

panel). The virtual absence of CNa₂S'' is due to the rapid dissociation ($k_{45} = 800 \text{ sec}^{-1}$) into CNa₂' and internal free sugar. CNa₂' and C' are only 8 and 2% of the carriers present. As result, when the potential is stepped from -50 to +50 mV, the clockwise reaction $\text{CNa}_2' \xrightarrow{k_{21}} \text{C}' \xrightarrow{k_{16}} \text{C}''$ is negligible. Our model predicts that carrier-mediated current transiently rises only to 94 nA, but relaxes to a steady current of +12 nA with a time constant τ of 13 msec (Fig. 9, lower panel). Presteady-state currents generated in the presence of saturating [αMDG]_o are thus 10-fold smaller than the presteady-state currents predicted solely in the presence of 100 mM [Na]_o. We think that the modest peak current and the noncompensated capacitive transient account for our failure to detect experimentally presteady-state currents under these conditions. As the potential is stepped back to -50 mV, an inward current develops that relaxes slowly back to -45 nA.

Finally, presteady-state currents were not observed in low external Na⁺ and [αMDG]_o (see Fig. 7D, Parent et al., 1991a). Simulations performed with 10 mM [Na]_o show that at the holding potential of -50 mV only 3% of all carriers are in the CNa₂' state compared to 75% in the presence of 100 mM [Na]_o. Under these conditions, k_{12} is only 15 sec⁻¹ as compared to 1458 sec⁻¹ in the presence of 100 mM [Na]_o. Free carriers are still preferentially distributed at the external surface (C' = 66%) at -50 mV, and as the potential is pulsed to +50 mV, C' is transformed to C'' mono-exponentially with a time constant τ of 4 msec. Again, this rapid decay of C' to C'' is likely to be masked experimentally by the oocyte-capacitive transient.

The time course of the carrier-mediated currents as predicted by our model has been reconstructed at -150 and +50 mV in the absence of the oocyte-capacitive transient (Fig. 10). The time course of the predicted carrier-mediated current with 100 mM [Na]_o is shown as a solid line. The carrier-mediated current predicted after the addition of 1 mM [αMDG]_o is shown as a dashed line. There is a significant transient carrier-mediated current only in the presence of 100 mM [Na]_o and at +50 mV (τ ≈ 13 msec). This current vanishes well before the steady-state electrical properties of the cotransporter were measured (at ≈75 msec). In contrast, there is a significant steady-state inward current at -150 mV as predicted after the addition of [αMDG]_o. The difference in the carrier-mediated current measured at steady state before (solid line) and after (dashed line) the addition of sugar to the bath is the sugar-dependent current. The model predicts that sugar-dependent currents measured at steady state will be negligible or close to zero at +50 mV and will increase as the membrane potential is made more negative (see Fig. 5).

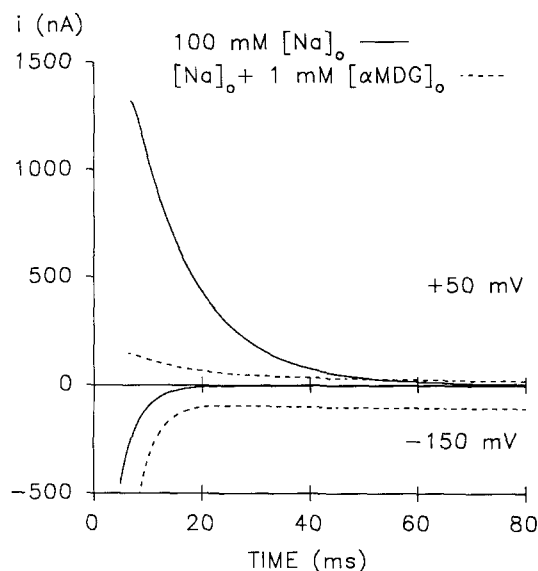


Fig. 10. Time course of the carrier-mediated current as predicted with the model. Carrier-mediated current is reconstructed in the absence of the oocyte-capacitive transient using Eq. (26) in the presence of 100 mM $[\text{Na}]_o$ (solid line) and after the addition of 1 mM $[\alpha\text{MDG}]_o$ to the bath (dashed line). Carrier-mediated current is shown as predicted at the test potential of +50 mV (upper panel) and -150 mV (lower panel). Presteady-state current peaks at 1320 nA at 6.75 msec in the presence of 100 mM $[\text{Na}]_o$ only. Sugar-dependent currents are the difference in current measured at 75 msec before and after the addition of sugar to the bath. At -150 mV, sugar-dependent currents are -105 nA. C_T was 6×10^{10} carriers oocyte⁻¹.

EVALUATION OF THE MODEL RATE CONSTANTS

The preceding description clearly illustrates how the characteristics of the presteady-state currents set strong limitations in the values of at least five rate constants k_{12} , k_{21} , k_{16} , k_{61} and k_{32} . Presteady-state currents can only be observed significantly over the oocyte-capacitive transient providing that CNa_2' relaxes to C'' with $\text{CNa}_2' \xrightarrow{k_{21}} \text{C}' \xrightarrow{k_{16}} \text{C}''$ at +50 mV. The relaxation time constant of the presteady-state current is determined largely by k_{16} ($\text{C}' \xrightarrow{k_{16}} \text{C}''$). Increasing k_{16} from 35 to only 70 sec⁻¹ would accelerate the relaxation of presteady-state currents up to the value of the oocyte-capacitive transient. The magnitude of the carrier peak transient current is controlled by the concentration of carrier state CNa_2' and C' before the depolarization pulse at any given C_T . Off-current or tail presteady-state currents are determined by k_{61} in the reverse reaction $\text{C}' \xrightarrow{k_{61}} \text{C}''$. The $[\text{Na}]_o$ dependence of the presteady-state currents requires that $k_{12}^*[\text{Na}]_o^\dagger$ must be such that at a

holding potential of -50 mV and in the absence of external sugar, CNa_2' would be the major carrier state with 100 mM $[\text{Na}]_o$ but not with 10 mM $[\text{Na}]_o$. Our failure to observe presteady-state currents in low external Na⁺ ions ($[\text{Na}]_o < 30$ mM) sets an upper limit on k_{12} ($k_{12} \ll 4 \times 10^5$). When k_{12} is increased beyond this limit, one may artificially generate "presteady-state currents" at any finite $[\text{Na}]_o$. Finally, the absence of detectable presteady-state currents in the presence of external sugar indicates that rate constant k_{32} must be lower than 25 sec⁻¹ so that the fully loaded carrier $\text{CNa}_2\text{S}'$ cannot reverse to $\text{CNa}_2' \xrightarrow{k_{21}} \text{C}' \xrightarrow{k_{16}} \text{C}''$. Simulations have shown that the magnitude of the predicted presteady-state currents decreases progressively as k_{32} is reduced from 20 to 1 sec⁻¹ in the presence of 1 mM $[\alpha\text{MDG}]_o$. It is concluded that the absolute values, and not only the ratio of these five rate constants, directly modulate presteady-state currents. Considering the requirement for external Na⁺- and sugar-association rate constants (k_{12} , k_{21} , k_{32}) and carrier-translocation rate constants (k_{16} and k_{61}), it is difficult to conceive that the carrier-mediated sugar transport can be described as a true rapid equilibrium system.

The voltage dependence of steady-state electrical properties $K_{0.5}$ and i_{max} is empirically described by the coefficients α' and δ , respectively.¹⁰ The fractional electrical coefficient for external Na⁺ binding, α' , characterizes the voltage dependence of $K_{0.5}$, and the fractional electrical coefficient for the carrier translocation, δ , characterizes the voltage dependence of i_{max} . As seen in the companion paper, both $K_{0.5}^{\text{Na}}$ and $K_{0.5}^{\alpha\text{MDG}}$, at low external $[\text{Na}]_o$, are voltage dependent. Their voltage dependence is best described by α' values between 0.25 and 0.35. Increasing α' beyond this range would steepen the voltage dependence of $K_{0.5}^{\alpha\text{MDG}}$ and $K_{0.5}^{\text{Na}}$. Voltage coefficient δ controls the sigmoidal shape of the I - V relationships and the Na dependence of $i_{\text{max}}^{\alpha\text{MDG}}$. Only δ values ≥ 0.7 can account for the sigmoidal shape and the Na dependence of $i_{\text{max}}^{\alpha\text{MDG}}$ curves. Smaller δ values tend to give I - V curves that are linear over the complete range of potential and larger external Na dependence than experimentally measured. The voltage dependence of $i_{\text{max}}^{\text{Na}}$ can be accounted for by δ values ranging from 0.4 to 0.8. Since by definition $\alpha' + \delta + \alpha'' = 1$, this leaves the coefficient α'' close to zero, suggesting a quasi-absence of voltage dependence for internal Na⁺ binding. Experiments from the internal face of the membrane are now required to determine the value of α'' and to examine the voltage dependence of internal Na⁺ binding.

[†] Rate constant k_{12}^* includes the effect of potential on the rate constant but not the effect of concentrations. See Appendix for further details.

¹⁰ The interpretation of the presteady-state currents relies on the assumption that the empty carrier is negatively charged but the model can still qualitatively account for the voltage dependence of $K_{0.5}^{\alpha\text{MDG}}$, $K_{0.5}^{\text{Na}}$, $i_{\text{max}}^{\text{Na}}$, and $i_{\text{max}}^{\alpha\text{MDG}}$ in the case where z is positive or equal to zero.

The Na⁺-dependence of $K_{0.5}^{\alpha\text{MDG}}$ and $i_{\text{max}}^{\alpha\text{MDG}}$ is a function of the rate constants k_{21} and k_{12} (whose ratio would be equivalent to $K\text{Na}'$). $K\text{Na}'$ values between 50 and 100 mM were found compatible with the experimental $K_{0.5}^{\text{Na}}$ values extrapolated at 0 mV. However, we found that increasing k_{12} , k_{21} , k_{32} , k_{23} , k_{56} , k_{65} , and k_{45} by a factor of 10 (thus conserving the same apparent $K\text{Na}'$, $K\text{Na}''$, and $K\text{Na}_2S'$ values), would overestimate the Na⁺ dependence of $K_{0.5}^{\alpha\text{MDG}}$ and reduce all the $K_{0.5}^{\text{Na}}$ values. This further suggests that k_{21} and k_{12} cannot be replaced by their equilibrium constant.

Rate constants k_{34} and k_{43} describe the membrane translocation of the fully loaded carrier $\text{CNa}_2S' \rightleftharpoons \text{CNa}_2S''$. In our model, k_{43} can be given any value between 0.1 and 1000 sec⁻¹ without affecting the presteady-state currents and the cotransporter steady-state kinetics $K_{0.5}^{\alpha\text{MDG}}$, $K_{0.5}^{\text{Na}}$, $i_{\text{max}}^{\alpha\text{MDG}}$ and $i_{\text{max}}^{\text{Na}}$. In contrast, k_{34} is only acceptable in a narrow range between 35 and 100 sec⁻¹. Decreasing k_{34} below 35 sec⁻¹ would overestimate $K_{0.5}^{\alpha\text{MDG}}$ and $K_{0.5}^{\text{Na}}$ while underestimating the current.¹¹ Increasing k_{34} above 100 sec⁻¹ would steepens the dependence of $i_{\text{max}}^{\alpha\text{MDG}}$ and $i_{\text{max}}^{\text{Na}}$ on membrane potential and significantly underestimate $K_{0.5}^{\alpha\text{MDG}}$ at low $[\text{Na}]_o$ and $K_{0.5}^{\text{Na}}$. In the 35- to 100-sec⁻¹ range, the variations of the steady-state parameters predicted by the model are within the error associated with the experimental results. Interestingly, the translocation rate constant of the loaded carrier (k_{34}) agrees with the Na⁺/glucose cotransporter turnover rates reported from 5 to 70 sec⁻¹ (see Peerce & Wright, 1984; Schultz, 1986).

The carrier-mediated Na⁺ leak current, $\text{CNa}_2' \rightleftharpoons \text{CNa}_2''$ translocation in the absence of sugar, is required to account for the observation of steady-state Na⁺-dependent and phlorizin-inhibited currents (Umbach et al., 1990; Parent et al., 1991a). The magnitude of the carrier "Na⁺ leakage" current is most sensitive to rate constants k_{25} and k_{52} . Increasing k_{25} and k_{52} increases the proportion of sugar-independent over sugar-dependent current. One should note, however, that the carrier Na⁺ leak current is not required to account for the presteady-state currents or to account for the carrier kinetics. Rate constants k_{25} and k_{52} can be set to 0.0 sec⁻¹ without affecting the adequateness of the model to $K_{0.5}^{\alpha\text{MDG}}$, $i_{\text{max}}^{\alpha\text{MDG}}$, $K_{0.5}^{\text{Na}}$, and $i_{\text{max}}^{\text{Na}}$ or the characteristics of the presteady-state currents (results not shown).

Rate constants k_{56} and k_{65} in $\text{C}'' \rightleftharpoons \text{CNa}_2''$ give the apparent internal Na dissociation constant $K\text{Na}''$. At any given k_{65} , increasing k_{56} from 1 to 50 sec⁻¹ will increase the current and steepen the voltage depen-

dence of $i_{\text{max}}^{\text{Na}}$ and $i_{\text{max}}^{\alpha\text{MDG}}$ so k_{56} can be set within a definite range. Rate constant k_{65} can be set anywhere between 20 and 1000 sec⁻¹ without any significant effect on $K_{0.5}^{\alpha\text{MDG}}$, $K_{0.5}^{\text{Na}}$, $i_{\text{max}}^{\alpha\text{MDG}}$, $i_{\text{max}}^{\text{Na}}$ predicted under zero-trans conditions. However, the range of trans-sugar inhibition effect is most sensitive to k_{65} . In the absence of experimental results performed under controlled trans conditions, k_{65} is yet to be narrowed down.

Dissociation constants for Na⁺ ions and sugar were computed. The external Na⁺ ions dissociation constant $K\text{Na}'$ ($K\text{Na}' = k_{21}/k_{12}$) is 79 mM at 0 mV and decreases to 24 mM as the membrane potential increases to -150 mV. The Na⁺ dissociation constant from the internal face ($K\text{Na}''$) is given by the ratio of the constants k_{56}/k_{65} . In the absence of a direct experimental result, any value of $K\text{Na}''$ between 40 and 1000 mM seems compatible with our results. The actual value may be inferred in the future from experiments measuring the Na-trans (Na_i) effect on sugar transport in *Xenopus* oocytes. The external sugar dissociation constant $K\text{Na}_2S'$ is 200 μM. Again, in the absence of direct experimental evidence, our results are compatible with any value of $K\text{Na}_2S''$ between 100 μM and 5 mM.

Model Predictions on TRANS Effects

Na-Trans INHIBITION ON THE COTRANSPORTER SUGAR-DEPENDENT CURRENT

Trans or internal Na⁺ ions inhibit sugar influx in BBMVs (see for instance, Kessler & Semenza, 1983; Kaunitz & Wright, 1984; Supplisson, 1988). In our model, the internal Na⁺ dissociation constant corresponds approximately to the half-maximum for Na-trans inhibition (K_i^{Na}) on the sugar-dependent currents. $K\text{Na}''$ values from 40 to 400 mM are compatible with our results with the maximum Na-trans inhibition effect at the lower end of the spectrum. With $K\text{Na}'' = 44$ mM, simulations shows that increasing $[\text{Na}]_i$ from 0 to 500 mM (in the absence of internal sugar) would progressively inhibit sugar-dependent currents (Fig. 11). Negative inside potential ($V_m < 0$ mV) would relieve Na-trans inhibition as reported by Kimmich and Randles (1988) in chicken enterocytes. With $K\text{Na}'' = 44$ mM, our model predicts a K_i for Na-trans inhibition of 44 mM at 0 mV, of 75 mM at -50 mV and of 350 mM at -150 mV. Our predicted K_i values agree with the K_i value of 54 mM reported at 0 mV by Kaunitz and Wright (1984).

¹¹ Any increase in the limiting rate constant of the cycle leads to an increase in the carrier-mediated current.

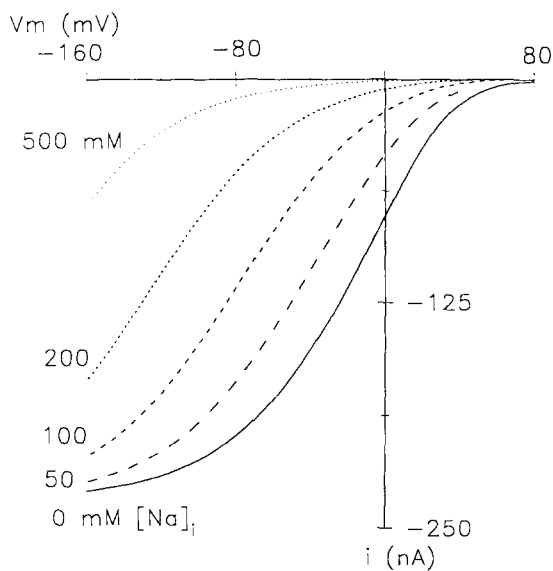


Fig. 11. *Trans*-Na inhibition on sugar-dependent currents as predicted by the model. Sugar-dependent *I*-*V* curves were simulated with 100 mM [Na]_o and 1 mM [αMDG]_o. Internal sugar was fixed at 0 μM. *Trans*-Na was varied between 0 and 500 mM. $KNa'' = 44$ mM, $k_{65} = 5000$ sec⁻¹ and $k_{56} = 10$ sec⁻¹. C_7 is 2×10^{11} carriers oocyte⁻¹.

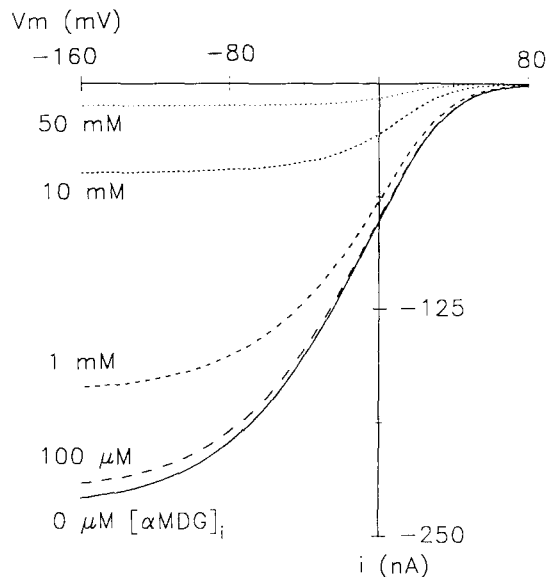


Fig. 12. *Trans*-sugar inhibition on sugar-dependent currents as predicted by the model. Sugar-dependent *I*-*V* curves were simulated with 100 mM [Na]_o and 1 mM [αMDG]_o. Internal [Na]_i was kept at 0 mM. *Trans*-sugar was varied from 0 to 50 mM. $KNaS'' = 4$ mM, $KNa' = 44$ mM, $k_{65} = 5000$ sec⁻¹ and $k_{56} = 10$ sec⁻¹. C_7 is 2×10^{11} carriers oocyte⁻¹.

SUGAR-*Trans* INHIBITION ON THE COTRANSPORTER SUGAR-DEPENDENT CURRENTS

Our model also predicts that *trans* sugar will inhibit sugar-dependent currents measured in the presence of external (*cis*) Na⁺ and sugar. The predicted midpoint of internal *trans*-sugar inhibition on currents is dependent upon the value attributed to KNa'' . With $KNa'' = 44$ mM, increasing internal sugar from 0 to 50 mM decreases sugar-dependent currents and membrane potential is not predicted to relieve the sugar-*trans* effect (Fig. 12). In our model, the magnitude of the *trans*-sugar effect is of course related to KNa'' . Increasing KNa'' toward the upper end of the spectrum would decrease the K_i for *trans*-sugar inhibition.¹² With $KNa'' = 450$ mM and $KNaS'' = 200$ μM, the major inhibition effect is found in the 50- to 500-μM range. Decreasing k_{65} to 5000 sec⁻¹, therefore decreasing KNa'' to 44 mM and $KNaS''$ to 4 mM, increases the midpoint of internal *trans*-sugar inhibition to 4 mM. Sugar-*trans* inhibition effect is predicted in the absence and in the presence of internal Na (no Na-*trans* relief effect). In addition, in-

¹² At fixed $k_{56} = 10$ sec⁻¹, increasing k_{65} from 50 to 5000 sec⁻¹ mol⁻² decreases KNa'' and increases $KNaS''$. In the absence of any result obtained under controlled *trans* conditions, KNa'' was giving values between 40 and 450 mM. $KNaS''$ varied accordingly from 4 mM to 200 μM.

creasing [S], above the value of $KNaS''$ is predicted to cause an increase in $K_{0,5}^{\alpha MDG}$ at low [Na]_o in the -50 to 0-mV range (*results not shown*). Kessler and Semenza (1983) have demonstrated sugar-*trans* inhibition in BBMV uptake experiments.

CONCLUSION

A 6-state ordered (2Na⁺:S) kinetic model was found to account for the presteady-state currents and the steady-state electrical properties of the Na⁺/glucose cotransporter. Our results are compatible with a negatively charged carrier with two Na⁺ binding sites located within the membrane electrical field ($\alpha' = 25$ to 35%). The Na⁺ ion binding sites on the empty carrier traverses 80 to 60% of the membrane electrical field during translocation. Negative membrane potential facilitates sugar transport. The preferential accumulation of the empty carrier at the outer membrane requires that the membrane potential be equal or more negative than -35 mV.

Presteady-state currents are described as gating currents of the Na⁺/glucose cotransporter. They are accounted for by the reaction steps $CNa_2' \xrightarrow{k_{21}} C' \xrightarrow{k_{16}} C''$ when the membrane potential is stepped from -50 to +50 mV in the presence of saturating [Na]_o. The relaxation time constant is function of both k_{21} and k_{16} , and the magnitude of the current is deter-

mined by the proportion of CNa_2' species available before the potential is stepped to +50 mV. Both ON and OFF currents are crucial in limiting the rate constants k_{12} , k_{21} , k_{16} , k_{61} and k_{32} . In our model, that means k_{21} should be between 200 and 600 sec⁻¹, k_{16} between 25 and 55 sec⁻¹, k_{61} between 3 and 5 sec⁻¹ and k_{32} between 1 and 30 sec⁻¹. Because of the presteady-state characteristics, k_{21} and k_{12} cannot be replaced by the equilibrium constant for external Na⁺ ions binding KNa' , and k_{32} , and k_{23} cannot be replaced by the equilibrium constant for external sugar $KNaS'$ which suggests that the rapid equilibrium assumption is improper in our conditions.

Rate-limiting steps for sugar transport are dependent upon the membrane potential and upon $[Na]_o$ and $[sugar]_o$. Under saturating $[Na]_o$ and $[S]_o$, the rate limiting for sugar transport is the outward carrier translocation $C' \xrightarrow{k_{61}} C'$ at any membrane potential more positive than -43 mV. In the membrane potential range between -43 and -134 mV, the rate-limiting step becomes the internal Na⁺ ions dissociation rate constant k_{56} . Since rapid equilibrium was not assumed in our model, k_{12} may be rate limiting at very low $[Na]_o$ (<10 mM) at negative membrane potential between -134 and 0 mV simply because the effect of membrane potential is stronger on k_{61} than on k_{12} .

The ratio of the empty carrier translocation $k_{16}/k_{61} = 7$, the ratio of the loaded carrier translocation $k_{34}/k_{43} = 1$, and the ratio of the loaded carrier translocation over the empty carrier translocation $k_{34}/k_{61} = 10$. This agrees with previous reports that J_{max} for the Na⁺/glucose cotransporter measured in BBMV under equilibrium exchange condition ($[Na]_o = [Na]_i$ and $[S]_o = [S]_i$) would be 5 to 10 times bigger than J_{max} measured under zero-*trans* conditions (see Stevens, Kaunitz & Wright, 1984).

The Na⁺/glucose cotransporter model is functionally asymmetrical with asymmetry in the rate constants for empty carrier translocation ($k_{61}^0 > k_{16}^0$), an absence of voltage dependence for internal Na⁺ binding ($\alpha'' = 0$), and internal Na⁺ binding rate constant k_{55}^0 200- to 2000-fold smaller than external Na⁺ binding k_{12}^0 ($KNa' = 80$ mM and $KNa'' = 44$ mM).

We are grateful to Drs. A. Berteloot, S. Ciani, and J.-Y. Lapointe for stimulating discussions and thank our colleagues for comments. L.P. was recipient of a post-doctoral fellowship from the Medical Research Council of Canada. This work was supported by a grant from the U.S. Public Health Service DK 19567.

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Received 12 April 1991; revised 25 June 1991

Appendix

The model for the Na⁺/glucose cotransporter involves 6 states (C_1, \dots, C_6) which contains two closed loops of 4 states each (Fig. A1).

DERIVATION OF THE STEADY-STATE CURRENT EQUATION FOR A 6-STATE ORDERED BINDING COTRANSPORT MODEL WITHOUT ASSUMING RAPID EQUILIBRIUM

Current equations were derived using the graphical method of King and Altman (1956). Figure A1 shows the 15 different King-Altman patterns.

Thus the steady-state concentration of each carrier form is solved according to:

$$\frac{[C_x]}{C_T} = \frac{\sum_{j=1}^{15} \text{KAT}_x^j}{\sum_{i=1}^6 \sum_{j=1}^{15} \text{KAT}_i^j} \quad (\text{A1})$$

where C_x ($x = 1, \dots, 6$) is a given state of the carrier and $C_T = \frac{\text{number of carriers per oocyte}}{6.022 \times 10^{23}} \cdot \text{KAT}_i^j$ is an individual King-Altman term with i being any carrier state and j one of the King-Altman patterns. Each term is the product of five pseudo first order rate constants (see Eqs. (A14) to (A19)).

INDIVIDUAL RATE CONSTANTS

Rapid equilibrium binding of Na⁺ ions and rapid equilibrium binding of sugar was not assumed. Every reaction step between two carrier states was treated as a function of first order ($k_{16}, k_{61}, k_{21}, k_{32}, k_{34}, k_{43}, k_{45}, k_{25}, k_{52}, k_{65}$) or pseudo first order rate constants

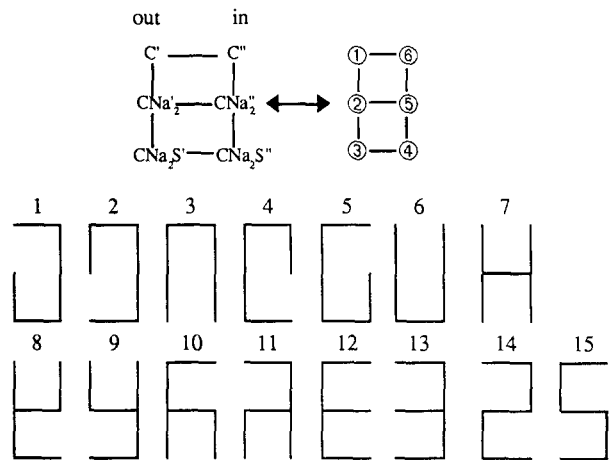


Fig. A1. Model, carrier states and the corresponding King-Altman patterns.

($k_{12}, k_{23}, k_{65}, k_{54}$). The effect of voltage and ligand concentration (Na⁺ ions or sugar) was accounted for in the calculation of the rate constants.

Voltage may influence both Na⁺ ion binding and carrier translocation. The effect of membrane potential on rate constants was treated according to the Eyring theory of transition states, assuming a single and symmetrical energy barrier for each reaction step. We used the terminology introduced by Lauger and Jauch (1986) where α' and α'' describe the fraction of the membrane field between Na⁺ ions and the Na⁺ binding sites and δ characterizes the effect of membrane potential on carrier translocation, and by definition $\alpha' + \delta + \alpha'' = 1$.

Pseudo first order rate constants (sec⁻¹) are described by:

$$k_{16} = k_{16}^0 e^{(-z_c \delta \frac{\mu}{2})} = k_{16}^0 e^{\delta \mu} \quad (\text{A2})$$

$$k_{61} = k_{61}^0 e^{(z_c \delta \frac{\mu}{2})} = k_{61}^0 e^{-\delta \mu} \quad (\text{A3})$$

$$k_{12} = k_{12}^0 [\text{Na}]_o^n e^{(-n z_{\text{Na}} \alpha' \frac{\mu}{2})} = k_{12}^0 [\text{Na}]_o^n e^{(-\alpha' \mu)} = k_{12}^* [\text{Na}]_o^2 \quad (\text{A4})$$

$$k_{21} = k_{21}^0 e^{(n z_{\text{Na}} \alpha' \frac{\mu}{2})} = k_{21}^0 e^{(\alpha' \mu)} \quad (\text{A5})$$

$$k_{65} = k_{65}^0 [\text{Na}]_i^n e^{(n z_{\text{Na}} \alpha' \frac{\mu}{2})} = k_{65}^0 [\text{Na}]_i^n e^{(\alpha' \mu)} = k_{65}^* [\text{Na}]_i^2 \quad (\text{A6})$$

$$k_{56} = k_{56}^0 e^{(-n z_{\text{Na}} \alpha' \frac{\mu}{2})} = k_{56}^0 e^{(-\alpha' \mu)} \quad (\text{A7})$$

$$k_{23} = k_{23}^0 [S]_o \quad (\text{A8})$$

$$k_{54} = k_{54}^0 [S]_i \quad (\text{A9})$$

where k_{12}^* and k_{65}^* include the effect of voltage and

$$\mu = \frac{FV}{RT} \quad (\text{A10})$$

where z_c is the valence of the ion binding site on the empty carrier ($z_c = -2$), n the apparent coupling coefficient ($n = 2$), z_{Na} the valence of sodium ions ($z_{\text{Na}} = 1$), V , R , T and F have their usual meanings.

MICROSCOPIC REVERSIBILITY

The thermodynamic requirement for a microscopic reversibility (no accumulation of carrier can occur in the absence of driving force) imposes a constraint for the determination of one rate constant on each cycle. In our simulation program, k_{52} and k_{64}^0 , were systematically calculated from the values of the other rate constants such as:

$$k_{52} = \frac{k_{12}^0 k_{23}^0 k_{34}^0 k_{61}^0}{k_{21}^0 k_{16}^0 k_{65}^0} \quad (\text{A11})$$

$$k_{64}^0 = \frac{k_{23}^0 k_{34}^0 k_{45}^0 k_{52}^0}{k_{43}^0 k_{32}^0 k_{25}^0} \quad (\text{A12})$$

STEADY-STATE CARRIER CONCENTRATIONS

The conservation equation for all carrier species is given by:

$$C_7 = [C]' + [\text{CNa}_2]' + [\text{CNa}_2S]' + [\text{CNa}_2S]'' + [\text{CNa}_2]'' + [C]'' \quad (\text{A13})$$

The steady-state concentration of each carrier state could be estimated using the King-Altman procedure shown in Fig. A1. For simplicity, the KAT terms are given in the form:

$$\sum_{j=1}^{15} \text{KAT}_x^j = \sum C_x$$

$$\begin{aligned} \sum C_1 = & k_{21}k_{32}k_{43}k_{52}k_{61} + k_{21}k_{32}k_{45}k_{52}k_{61} + k_{21}k_{34}k_{45}k_{52}k_{61} \\ & + k_{21}k_{32}k_{43}k_{54}k_{61} + k_{21}k_{32}k_{43}k_{56}k_{61} + k_{25}k_{32}k_{43}k_{56}k_{61} \\ & + k_{21}k_{32}k_{45}k_{56}k_{61} + k_{25}k_{32}k_{45}k_{56}k_{61} + k_{21}k_{34}k_{45}k_{56}k_{61} \\ & + k_{23}k_{34}k_{45}k_{56}k_{61} + k_{25}k_{34}k_{45}k_{56}k_{61} + k_{21}k_{32}k_{43}k_{52}k_{65} \\ & + k_{21}k_{32}k_{45}k_{52}k_{65} + k_{21}k_{34}k_{45}k_{52}k_{65} + k_{21}k_{32}k_{43}k_{54}k_{65} \end{aligned} \quad (\text{A14})$$

$$\begin{aligned} \sum C_2 = & k_{12}k_{32}k_{43}k_{52}k_{61} + k_{12}k_{32}k_{45}k_{52}k_{61} + k_{12}k_{34}k_{45}k_{52}k_{61} \\ & + k_{12}k_{32}k_{43}k_{54}k_{61} + k_{12}k_{32}k_{43}k_{56}k_{61} + k_{12}k_{32}k_{45}k_{56}k_{61} \\ & + k_{12}k_{34}k_{45}k_{56}k_{61} + k_{12}k_{32}k_{43}k_{52}k_{65} + k_{16}k_{32}k_{43}k_{52}k_{65} \\ & + k_{12}k_{32}k_{45}k_{52}k_{65} + k_{16}k_{32}k_{45}k_{52}k_{65} + k_{12}k_{34}k_{45}k_{52}k_{65} \\ & + k_{16}k_{34}k_{45}k_{52}k_{65} + k_{12}k_{32}k_{43}k_{54}k_{65} + k_{16}k_{32}k_{43}k_{54}k_{65} \end{aligned} \quad (\text{A15})$$

$$\begin{aligned} \sum C_3 = & k_{12}k_{23}k_{43}k_{52}k_{61} + k_{12}k_{23}k_{45}k_{52}k_{61} + k_{12}k_{23}k_{43}k_{54}k_{61} \\ & + k_{12}k_{25}k_{43}k_{54}k_{61} + k_{12}k_{23}k_{43}k_{56}k_{61} + k_{12}k_{23}k_{43}k_{56}k_{61} \\ & + k_{12}k_{23}k_{43}k_{52}k_{65} + k_{16}k_{23}k_{43}k_{52}k_{65} + k_{12}k_{23}k_{45}k_{52}k_{65} \\ & + k_{16}k_{23}k_{45}k_{52}k_{65} + k_{16}k_{21}k_{43}k_{54}k_{65} + k_{12}k_{23}k_{43}k_{54}k_{65} \\ & + k_{16}k_{23}k_{43}k_{54}k_{65} + k_{12}k_{25}k_{43}k_{54}k_{65} + k_{16}k_{25}k_{43}k_{54}k_{65} \end{aligned} \quad (\text{A16})$$

$$\begin{aligned} \sum C_4 = & k_{12}k_{23}k_{34}k_{52}k_{61} + k_{12}k_{25}k_{32}k_{54}k_{61} + k_{12}k_{23}k_{34}k_{54}k_{61} \\ & + k_{12}k_{25}k_{34}k_{54}k_{61} + k_{12}k_{23}k_{34}k_{56}k_{61} + k_{12}k_{23}k_{34}k_{52}k_{65} \\ & + k_{16}k_{23}k_{34}k_{52}k_{65} + k_{16}k_{21}k_{32}k_{54}k_{65} + k_{12}k_{25}k_{32}k_{54}k_{65} \\ & + k_{16}k_{25}k_{32}k_{54}k_{65} + k_{16}k_{21}k_{34}k_{54}k_{65} + k_{12}k_{23}k_{34}k_{54}k_{65} \\ & + k_{16}k_{23}k_{34}k_{54}k_{65} + k_{12}k_{25}k_{34}k_{54}k_{65} + k_{16}k_{25}k_{34}k_{54}k_{65} \end{aligned} \quad (\text{A17})$$

$$\begin{aligned} \sum C_5 = & k_{12}k_{25}k_{32}k_{43}k_{61} + k_{12}k_{25}k_{32}k_{45}k_{61} + k_{12}k_{23}k_{34}k_{45}k_{61} \\ & + k_{12}k_{25}k_{34}k_{45}k_{61} + k_{16}k_{21}k_{32}k_{43}k_{65} + k_{12}k_{25}k_{32}k_{43}k_{65} \\ & + k_{16}k_{25}k_{32}k_{43}k_{65} + k_{16}k_{21}k_{32}k_{45}k_{65} + k_{12}k_{25}k_{32}k_{45}k_{65} \\ & + k_{16}k_{25}k_{32}k_{45}k_{65} + k_{16}k_{21}k_{34}k_{45}k_{65} + k_{12}k_{23}k_{34}k_{45}k_{65} \\ & + k_{16}k_{23}k_{34}k_{45}k_{65} + k_{12}k_{25}k_{34}k_{45}k_{65} + k_{16}k_{25}k_{34}k_{45}k_{65} \end{aligned} \quad (\text{A18})$$

$$\begin{aligned} \sum C_6 = & k_{16}k_{21}k_{32}k_{43}k_{52} + k_{16}k_{21}k_{32}k_{45}k_{52} + k_{16}k_{21}k_{34}k_{45}k_{52} \\ & + k_{16}k_{21}k_{32}k_{43}k_{54} + k_{16}k_{21}k_{32}k_{43}k_{56} + k_{12}k_{25}k_{32}k_{43}k_{56} \\ & + k_{16}k_{25}k_{32}k_{43}k_{56} + k_{16}k_{21}k_{32}k_{45}k_{56} + k_{12}k_{25}k_{32}k_{45}k_{56} \\ & + k_{16}k_{25}k_{32}k_{45}k_{56} + k_{16}k_{21}k_{34}k_{45}k_{56} + k_{12}k_{23}k_{34}k_{45}k_{56} \\ & + k_{16}k_{23}k_{34}k_{45}k_{56} + k_{12}k_{25}k_{34}k_{45}k_{56} + k_{16}k_{25}k_{34}k_{45}k_{56} \end{aligned} \quad (\text{A19})$$

STEADY-STATE CURRENT EQUATION

The steady-state current equation is expressed as:

$$i = -z_c F (k_{16}[C]' - k_{61}[C]''). \quad (\text{A20})$$

Equation (A20) was then solved for $[C]'$ and $[C]''$ using Eqs. (A1) and (A14) to (A19). The steady-state current equation can be rearranged both for zero-*trans* and nonzero-*trans* conditions as a function of the external sodium and sugar concentrations with $z_c = -2$ and $n = 2$:

$$i = 2FC_7 \left[\frac{\varepsilon[\text{Na}]_o^2[S]_o + \phi[\text{Na}]_o^2 + \gamma}{\alpha + \beta[S]_o + \chi[\text{Na}]_o^2 + [\text{Na}]_o^2[S]_o} \right]. \quad (\text{A21})$$

Under nonzero-*trans* conditions ($[\text{Na}]_i > 0$ and $[S]_i > 0$), the macro constants are defined by:

$$\varepsilon = -\frac{1}{\lambda} k_{12}^* k_{23}^0 k_{34}^0 k_{45}^0 k_{56}^0 k_{61} \quad (\text{A22})$$

$$\phi = -\frac{1}{\lambda} k_{12}^* k_{56}^0 k_{61} (k_{25} k_{34} k_{45} + k_{32} k_{25} k_{43} + k_{32} k_{25} k_{45}) \quad (\text{A23})$$

$$\gamma = \frac{1}{\lambda} k_{16} k_{21} k_{65} (k_{34} k_{45} k_{52} + k_{32} k_{43} k_{52} + k_{32} k_{45} k_{52} + k_{32} k_{43} k_{54}) \quad (\text{A24})$$

$$\alpha = \frac{1}{\lambda} (k_{16} k_{21} k_{32} k_{43} k_{52} + k_{16} k_{21} k_{32} k_{45} k_{52} + k_{16} k_{21} k_{34} k_{45} k_{52}$$

$$+ k_{16} k_{21} k_{32} k_{43} k_{54} + k_{16} k_{21} k_{32} k_{43} k_{56} + k_{16} k_{25} k_{32} k_{43} k_{56}$$

$$+ k_{16} k_{21} k_{32} k_{45} k_{56} + k_{16} k_{25} k_{32} k_{45} k_{56} + k_{16} k_{21} k_{34} k_{45} k_{56}$$

$$+ k_{16} k_{25} k_{34} k_{45} k_{56} + k_{21} k_{32} k_{43} k_{52} k_{61} + k_{21} k_{32} k_{45} k_{52} k_{61}$$

$$+ k_{21} k_{34} k_{45} k_{52} k_{61} + k_{21} k_{32} k_{43} k_{54} k_{61} + k_{21} k_{32} k_{43} k_{56} k_{61}$$

$$+ k_{25} k_{32} k_{43} k_{56} k_{61} + k_{21} k_{32} k_{45} k_{56} k_{61} + k_{25} k_{32} k_{45} k_{56} k_{61}$$

$$+ k_{21} k_{34} k_{45} k_{56} k_{61} + k_{25} k_{34} k_{45} k_{56} k_{61} + k_{16} k_{21} k_{32} k_{43} k_{65}$$

$$+ k_{16} k_{25} k_{32} k_{43} k_{65} + k_{16} k_{21} k_{32} k_{45} k_{65} + k_{16} k_{25} k_{32} k_{45} k_{65}$$

$$+ k_{16} k_{21} k_{34} k_{45} k_{65} + k_{16} k_{25} k_{34} k_{45} k_{65} + k_{16} k_{32} k_{43} k_{52} k_{65}$$

$$\begin{aligned}
& + k_{21}k_{32}k_{43}k_{52}k_{65} + k_{16}k_{32}k_{45}k_{52}k_{65} + k_{21}k_{32}k_{45}k_{52}k_{65} \\
& + k_{16}k_{34}k_{45}k_{52}k_{65} + k_{21}k_{34}k_{45}k_{52}k_{65} + k_{16}k_{21}k_{32}k_{54}k_{65} \\
& + k_{16}k_{25}k_{32}k_{54}k_{65} + k_{16}k_{21}k_{34}k_{54}k_{65} + k_{16}k_{25}k_{34}k_{54}k_{65} \\
& + k_{16}k_{21}k_{43}k_{54}k_{65} + k_{16}k_{25}k_{43}k_{54}k_{65} + k_{16}k_{32}k_{43}k_{54}k_{65} \\
& + k_{21}k_{32}k_{43}k_{54}k_{65}
\end{aligned} \quad (A25)$$

$$\begin{aligned}
\beta = \frac{1}{\lambda} k_{23}^0 (k_{16}k_{34}k_{45}k_{56} + k_{34}k_{45}k_{56}k_{61} + k_{16}k_{34}k_{45}k_{65} + k_{16}k_{34}k_{52}k_{65} \\
+ k_{16}k_{43}k_{52}k_{65} + k_{16}k_{45}k_{52}k_{65} + k_{16}k_{34}k_{54}k_{65} + k_{16}k_{43}k_{54}k_{65})
\end{aligned} \quad (A26)$$

$$\begin{aligned}
\chi = \frac{1}{\lambda} k_{12}^* (k_{25}k_{32}k_{43}k_{56} + k_{25}k_{32}k_{45}k_{56} + k_{25}k_{34}k_{45}k_{56} + k_{25}k_{32}k_{43}k_{61} \\
+ k_{25}k_{32}k_{45}k_{61} + k_{25}k_{34}k_{45}k_{61} + k_{32}k_{43}k_{52}k_{61} + k_{32}k_{45}k_{52}k_{61} \\
+ k_{34}k_{45}k_{52}k_{61} + k_{25}k_{32}k_{54}k_{61} + k_{25}k_{34}k_{54}k_{61} + k_{25}k_{43}k_{54}k_{61} \\
+ k_{32}k_{43}k_{54}k_{61} + k_{32}k_{43}k_{56}k_{61} + k_{32}k_{45}k_{56}k_{61} + k_{34}k_{45}k_{56}k_{61} \\
+ k_{25}k_{32}k_{43}k_{65} + k_{25}k_{32}k_{45}k_{65} + k_{25}k_{34}k_{45}k_{65} + k_{32}k_{43}k_{52}k_{65} \\
+ k_{32}k_{45}k_{52}k_{65} + k_{34}k_{45}k_{52}k_{65} + k_{25}k_{32}k_{54}k_{65} + k_{25}k_{34}k_{54}k_{65} \\
+ k_{25}k_{43}k_{54}k_{65} + k_{32}k_{43}k_{54}k_{65})
\end{aligned} \quad (A27)$$

$$\begin{aligned}
\lambda = k_{12}^* k_{23}^0 (k_{34}k_{45}k_{56} + k_{34}k_{45}k_{61} + k_{34}k_{52}k_{61} + k_{43}k_{52}k_{61} + k_{45}k_{52}k_{61} \\
+ k_{34}k_{54}k_{61} + k_{43}k_{54}k_{61} + k_{34}k_{56}k_{61} + k_{43}k_{56}k_{61} + k_{45}k_{56}k_{61} \\
+ k_{34}k_{45}k_{65} + k_{34}k_{52}k_{65} + k_{43}k_{52}k_{65} + k_{45}k_{52}k_{65} + k_{34}k_{54}k_{65} \\
+ k_{43}k_{54}k_{65}).
\end{aligned} \quad (A28)$$

Assuming zero-*trans* conditions, ($[\text{Na}]_i = 0$, $[\text{S}]_i$, i.e. k_{65} and $k_{54} = 0$), the macro constants simplify further to:

$$\varepsilon = -\frac{1}{\lambda} k_{12}^* k_{23}^0 k_{34}k_{45}k_{56}k_{61} \quad (A29)$$

$$\phi = -\frac{1}{\lambda} k_{12}^* k_{25}k_{56}k_{61}(k_{32}k_{43} + k_{32}k_{45} + k_{34}k_{45}) \quad (A30)$$

$$\gamma = 0 \quad (A31)$$

$$\alpha = \frac{1}{\lambda} (k_{16} + k_{61})(k_{32}k_{43} + k_{32}k_{45} + k_{34}k_{45})(k_{21}k_{52} + k_{21}k_{56} + k_{25}k_{56}) \quad (A32)$$

$$\beta = \frac{1}{\lambda} k_{23}^0 k_{34}k_{45}k_{56}(k_{16} + k_{61}) \quad (A33)$$

$$\chi = \frac{1}{\lambda} k_{12}^* (k_{32}k_{43} + k_{32}k_{45} + k_{34}k_{45})(k_{25}k_{56} + k_{25}k_{61} + k_{52}k_{61} + k_{56}k_{61}) \quad (A34)$$

$$\begin{aligned}
\lambda = k_{12}^* k_{23}^0 (k_{34}k_{45}k_{56} + k_{34}k_{45}k_{61} + k_{34}k_{52}k_{61} + k_{43}k_{52}k_{61} \\
+ k_{45}k_{52}k_{61} + k_{34}k_{56}k_{61} + k_{43}k_{56}k_{61} + k_{45}k_{56}k_{61}).
\end{aligned} \quad (A35)$$

PHENOMENOLOGICAL CURRENT EQUATIONS AND CONSTANTS

Steady-state current Eq. (A21) can be rearranged as a sum of two phenomenological equations, as a function of either the external sugar, Eq. (A36), or the external sodium, Eq. (A40), concentration:

$$i = \left[\frac{I_{\max}^S [S]_o}{K_{0.5}^S + [S]_o} \right] + \left[\frac{I_{\max}^{S=0}}{K_{0.5}^S + [S]_o} \right] \quad (A36)$$

$$\left\{ \begin{aligned} I_{\max}^S &= 2FC_T \left[\frac{\varepsilon [\text{Na}]_o^2}{\beta + [\text{Na}]_o^2} \right] \end{aligned} \right. \quad (A37)$$

$$\left\{ \begin{aligned} I_{\max}^{S=0} &= 2FC_T \left[\frac{\phi [\text{Na}]_o^2 + \gamma}{\beta + [\text{Na}]_o^2} \right] \end{aligned} \right. \quad (A38)$$

$$\left\{ \begin{aligned} K_{0.5}^S &= \left[\frac{\alpha + \chi [\text{Na}]_o^2}{\beta + [\text{Na}]_o^2} \right] \end{aligned} \right. \quad (A39)$$

where ε , ϕ , γ , α , β , and χ had been previously defined by Eqs. (A22) to (A35)

$$i = \left[\frac{I_{\max}^{\text{Na}} [\text{Na}]_o^2}{[K_{0.5}^{\text{Na}}]^2 + [\text{Na}]_o^2} \right] + \left[\frac{I_{\max}^{\text{Na}=0}}{[K_{0.5}^{\text{Na}}]^2 + [\text{Na}]_o^2} \right] \quad (A40)$$

$$\left\{ \begin{aligned} I_{\max}^{\text{Na}} &= 2FC_T \left[\frac{\phi + \varepsilon [S]_o}{\chi + [S]_o} \right] \end{aligned} \right. \quad (A41)$$

$$\left\{ \begin{aligned} I_{\max}^{\text{Na}=0} &= 2FC_T \left[\frac{\gamma}{\chi + [S]_o} \right] \end{aligned} \right. \quad (A42)$$

$$\left\{ \begin{aligned} K_{0.5}^{\text{Na}} &= \left[\frac{\alpha + \beta [S]_o}{\chi + [S]_o} \right]. \end{aligned} \right. \quad (A43)$$

Phenomenological constants I_{\max}^S and $K_{0.5}^S$ were generated from Eqs. (A38) and (A40), and phenomenological constants I_{\max}^{Na} and $K_{0.5}^{\text{Na}}$ were generated from Eqs. (A41) and (A43). Under our experimental conditions $I_{\max}^{\text{Na}=0}$ and $I_{\max}^{S=0}$ were found to be negligible, which is confirmed when their value is calculated using the rate constants shown in Fig. 2.

PRESTEADY-STATE CURRENT: NUMERICAL INTEGRATION PROCEDURE

The time course of the carrier current in response to a step of voltage ΔV from a holding potential was estimated by numerical integration. The initial concentration of each carrier species was calculated according to Eqs. (A14) to (A19), assuming that the carrier distribution was at steady state at the holding potential ($t = 0$) just before the pulse of voltage. The time course of each carrier species was obtained by numerical integration of the following set of differential equations (A44) to (A49) with a time interval Δt between 10 and 50 μsec . The carrier current was calculated according to Eq. (A50).

$$\frac{d[C]'}{dt} = k_{61}[C]' + k_{21}[\text{CNa}_2]' - (k_{16} + k_{12})[C]' \quad (A44)$$

$$\begin{aligned}
\frac{d[\text{CNa}_2]'}{dt} &= k_{12}[C]' + k_{52}[\text{CNa}_2]'' + k_{32}[\text{CNa}_2S]' \\
&- (k_{21} + k_{25} + k_{23})[\text{CNa}_2]'
\end{aligned} \quad (A45)$$

$$\begin{aligned}
\frac{d[\text{CNa}_2S]'}{dt} &= k_{23}[\text{CNa}_2]' + k_{43}[\text{CNa}_2S]'' - (k_{32} + k_{34})[\text{CNa}_2S]' \\
&
\end{aligned} \quad (A46)$$

$$\begin{aligned}
\frac{d[\text{CNa}_2S]''}{dt} &= k_{34}[\text{CNa}_2S]' + k_{54}[\text{CNa}_2]'' - (k_{43} + k_{45})[\text{CNa}_2S]'' \\
&
\end{aligned} \quad (A47)$$

$$\begin{aligned}
\frac{d[\text{CNa}_2]''}{dt} &= k_{25}[\text{CNa}_2]' + k_{45}[\text{CNa}_2S]'' + k_{65}[C]' \\
&- (k_{52} + k_{54} + k_{56})[\text{CNa}_2]''
\end{aligned} \quad (A48)$$

$$\frac{d[C]''}{dt} = k_{16}[C]' + k_{56}[\text{CNa}_2]'' - (k_{61} + k_{65})[C]'' \quad (A49)$$

The presteady-state current is given by the sum of all translocation steps involving the transfer of a net charge in the membrane:

$$\begin{aligned}
i &= -F[z_{\text{Na}}\alpha'(k_{12}[C]' - k_{21}[\text{CNa}_2]') + z_c\delta'(k_{16}[C]' - k_{61}[C]') \\
&+ z_{\text{Na}}\alpha''(k_{56}[\text{CNa}_2]'' - k_{65}[C]'')].
\end{aligned} \quad (A50)$$